Option Pricing in the Cross-Section of Stock Returns

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Abstract

This paper explicitly derives the cross-sectional predictions of an intertemporal equilibrium asset pricing model when aggregate consumption volatility is stochastic and investors have aversion to both risk and expected downside losses. We show that an asset risk premium is not only determined by covariation of returns with the market return and changes in market volatility, but also by covariation of returns with three option payoffs: a cash-or-nothing option, a put option on the market return and a call option on changes in market volatility. These options provide a straightforward way for investors to act on their views of two of the most closely followed market variables, the market return and changes in market volatility. We show that the cross-section of stock returns reflects a premium for bearing undesirable exposures to these options, which we also show are rational interpretations for downside risks. The model provides a unified framework for understanding the various channels through which downside risks may affect asset prices. Our empirical results shed light on the total and relative economic significance of these channels.

Preliminary and incomplete. Please, do not circulate.

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1 Introduction

This paper explicitly derives the cross-sectional predictions of an intertemporal equilibrium asset pricing model when aggregate consumption volatility is stochastic and investors have aversion to both risk and expected downside losses. We show that an asset risk premium is not only determined by covariation of returns with the market return and changes in market volatility, but also by covariation of returns with three option payoffs: a cash-or-nothing option, a put option on the market return and a call option on changes in market volatility. These options provide a straightforward way for investors to act on their views of two of the most closely followed market variables, the market return and changes in market volatility. We show that the cross-section of stock returns reflects a premium for bearing undesirable exposures to these options, which we also show are rational interpretations for downside risks.

Standard capital asset pricing models all suggest that an asset premium is a compensation for the asset risk, where risk is understood as the covariance between the asset payoffs and the unexpected variations in priced factors at the marketplace. Examples of these factors are the market return in the CAPM of Sharpe (1964) and Lintner (1965), and fundamentals such as the consumption growth in the C-CAPM of Lucas (1978) and Breeden (1979), and both the consumption growth and the welfare valuation ratio growth in the recursive utility model of Epstein and Zin (1989) based on Kreps and Porteus (1978) preferences. Common to these models is that the investor has an equal treatment of risk across disappointing and satisfying market conditions, where disappointing market conditions correspond to periods where the market return or the growth in fundamentals falls below a reference threshold.

The asymmetric nature and treatment of risk has long been well-accepted among practitioners and academic researchers (Roy 1952; Markowitz 1959), and recently has led to new developments in asset pricing and financial risk management, such as the concept of the value-at-risk and the expected shortfall, as well as axiomatic approaches to preferences that allow investors to place greater weights on disappointing market conditions in their utility functions. These developments include the lower-partial moment framework of Bawa and Lindenberg (1977), the prospect theory of choice of Kahneman and Tversky (1979), and more recently the theory of disappointment aversion of Gul (1991), generalized by Routledge and Zin (2010) who embed them in the recursive utility framework of Epstein and Zin (1989). These new theories suggest priced downside risks in the capital market equilibrium.

Our study builds on generalized disappointment aversion (GDA) preferences. The disappointing event ($\mathcal{D}$) is endogenous to the model and corresponds to a situation where the market return sufficiently falls and/or changes in market volatility sufficiently increase. The GDA investor exhibits both risk aversion (i.e. aversion to regular betas on market return and on changes in market volatility) and disappointment aversion (i.e. aversion to expected downside losses). We refer to the combination of both risk and disappointment as the effective risk. We explicitly disentangle the components of the asset effective risk premium that are due to risk exclusively, from those that are due to disappointment exclusively, and from those that are due to the interaction between risk and disappointment.

An investor with expected utility (henceforth EU) preferences requires two premiums to invest in a risky asset. These two premiums are compensations for covariations of the asset payoff with the market return, $Cov\left(R_i^e, r_W\right)$, and with the changes in market volatility, $Cov\left(R_i^e, \Delta\sigma_W^2\right)$. These two premiums are exclusively due to risk aversion, since they are the only premiums required by a risk averse but disappointment neutral investor. The GDA investor requires three additional premiums. The first premium is a compensation for the covariance with the payoff of a binary cash-or-nothing option, $Cov\left(R_i^e, I\left(\mathcal{D}\right)\right)$, where $I\left(\cdot\right)$ is the indicator function that takes the value 1 if the condition is met and 0 otherwise. We show that this premium is exclusively due to disappointment aversion, since it is the only premium required by a risk neutral but disappointment averse investor. The second premium is a compensation for the covariance of the asset returns with a put option on the market return, $Cov\left(R_i^e, r_W I\left(\mathcal{D}\right)\right)$, and the third premium is a compensation for the covariance with a call option on changes in market volatility, $Cov\left(R_i^e, \Delta\sigma_W^2 I\left(\mathcal{D}\right)\right)$. These latter compensations are not exclusively due to either risk aversion or disappointment aversion, as they are required if and only if the investor is both risk averse and disappointment averse.

We explore the cross-sectional predictions of the model using all common stocks traded on the NYSE, AMEX and NASDAQ markets covering the period from July 1963 to December 2010.

The main results of the paper relate to the cross-sectional pricing of options on the market return and on changes in market volatility. Our empirical methodology uses portfolio sorts on individual stock exposures to these options, controlling for exposures to the market return and to changes in market volatility. Across individual stocks, we see a wide dispersion in sensitivity to options, which generates cross-sectional variation in the risk premia attributed to these factors. We further use cross-sectional regressions of Fama and MacBeth (1973) to estimate these factor risk premia. Our main finding is that options on the market return and on changes in the market volatility are highly significant factors in the cross-section of stock returns.

The estimated signs and magnitudes of factor risk premia associated with options on the market return and on changes in market volatility are all consistent with the theoretical implications. The put option has a positive risk premium. Assets that covary positively with the put option are undesirable because they tend to have low payoffs when an already low market return gets worse. In economic terms, our estimates suggest that a well-diversified single exposure to the put option on the market return has an annualized Sharpe ratio of 1.09 on average.

We also find that the cash-or-nothing option has a negative risk premium. An asset that covaries negatively with the cash-or-nothing option is undesirable because it has lower expected payoffs than usual when disappointment sets in, that is an asset with a low relative downside potential. We estimate that a well-diversified single exposure to the cash-or-nothing option yields an annualized Sharpe ratio of 0.51 on average. Finally, the call option carries a negative risk premium. Assets that covary negatively with the call option are undesirable because they tend to have low payoffs when an unusually high market volatility level further increases. Interpreting the estimated premium shows that a well-diversified single exposure to the call option on changes in market volatility has an annualized Sharpe ratio of 0.86 on average.

We complement the existing theoretical and empirical asset pricing literature on how asset prices are affected by downside risks. Practically, downside risks can be assessed through downside betas. These downside betas measure the comovements between asset payoffs and priced factors conditional upon disappointing market conditions, similar to the market downside beta, measured empirically and examined in the cross-section of stock returns by Ang, Chen and Xing (2006). In our setting, downside risks are interpretable as exposures to the put option on the market return and to the call option on changes in market volatility. We explicitly derive the market downside beta in terms of exposures to the market return, to changes in market volatility and to these two options.

This paper also touches in particular on the recent literature on systemic risk. Brownlees and Engle (2011) and Acharya et al. (2010) propose to measure systemic risk through the marginal expected shortfall, which they estimate empirically and examine for the regulation of systemic risk in US financial firms. In our setting, we interpret the exposure to the cash-or-nothing option as the relative downside potential of the asset and show how it relates to the marginal expected shortfall. Thus, instead of the regulation, we focus on the pricing of systemic risk. Furthermore, being motivated by dynamic consumption-based equilibrium asset pricing and behaviorial decision theory, our setup attempts to extend research on systemic financial risk onto many of the directions advocated by Brunnermeier et al. (2010).

Ultimately, a dynamic asset pricing model with asymmetric preferences delivers a unified theoretical framework that can explain the empirical findings that asset sensitivities to the market return and to changes in market volatility are priced (Ang, Hodrick, Xing and Zhang 2006; Adrian and Rosenberg 2008), that the market downside beta is priced (Ang, Chen and Xing 2006; Hong et al. 2006), and that the volatility downside beta and the relative downside potential of an asset are priced. There is little or no empirical evidence regarding the two latter measures, and we view this as an important contribution to the literature.

We also examine the empirical performance of our cross-sectional model on standard sets of sorted portfolios: size, book-to-market, momentum and industry portfolios. Our results still compare to those obtain on individual stocks. In terms of the pricing errors, our five-factor model with market beta, volatility beta and exposures to the three options provides a significant improvement over the standard CAPM model. It is comparable to the four-factor model of Carhart (1997), but in contrast, it has the benefit of being motivated by dynamic consumption-based equilibrium asset pricing and behaviorial decision theory.

We decompose the portfolio premia into parts attributable to each of the five factors from the model. We find that the three options account for non negligible parts of the total premium required to invest in stocks, and that they are relevant for interpreting differences in risk compensation across size, book-to-market, momentum and industry portfolios. We finally show that our results are robust to different data subsamples, to alternative measures of market volatility and to alternative definitions of the disappointment region.

The balance of the paper is organized as follows. In Section 2, we present and develop the theoretical setup from which we derive the implied cross-sectional model. Section 3 quantifies the factor premia in a calibrated consumption-based setting. Section 4 contains the empirical assessment of the model, while Section 5 describes how different measures of downside risk are related to our model. Section 6 concludes. An appendix contains additional material and proofs.

2 Theoretical Setup

In this section, we derive the cross-sectional implications of an asset pricing model where the representative agent has recursive utility with asymmetric preferences over disappointing versus satisfying economic situations.

2.1 Assumptions on Investors' Preferences

We consider a representative investor with generalized disappointment aversion preferences (GDA) of Routledge and Zin (2010). Following Epstein and Zin (1989) and Weil (1989), such an investor derives utility from consumption, recursively as follows:

$$\begin{aligned} V_t &= \left\{ (1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \\ &= C_t^{1-\delta} \left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{\delta} \quad \text{if } \psi = 1. \end{aligned} \tag{2}$$

The current period lifetime utility V_t is a combination of current consumption C_t, and $\mathcal{R}_t\left(V_{t+1}\right)$, a certainty equivalent of next period lifetime utility. With GDA preferences the risk-adjustment function $\mathcal{R}(.)$ is implicity defined by:

$$\frac{\mathcal{R}^{1-\gamma}-1}{1-\gamma}=\int_{-\infty}^{\infty}\frac{V^{1-\gamma}-1}{1-\gamma}dF(V)-\left(\frac{1}{\alpha}-1\right)\int_{-\infty}^{\kappa\mathcal{R}}\left(\frac{(\kappa\mathcal{R})^{1-\gamma}-1}{1-\gamma}-\frac{V^{1-\gamma}-1}{1-\gamma}\right)dF(V), \quad (3)$$

where $0<\alpha\leq 1$ and $0<\kappa\leq 1$. When α is equal to one, $\mathcal{R}$ reduces to expected utility (EU) preferences, while V_t represents the Epstein and Zin (1989) recursive utility. When $\alpha<1$, outcomes lower than $\kappa\mathcal{R}$ receive an extra weight, decreasing the certainty equivalent. Thus, the parameter α is interpreted as a measure of disappointment aversion, while the parameter κ is the percentage of the certainty equivalent $\mathcal{R}$ such that outcomes below it are considered disappointing[1].

With EU preferences, Hansen et al. (2008) derive the stochastic discount factor in terms of the continuation value of utility of consumption, as follows:

$$M^*_{t,t+1}=\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\frac{1}{\psi}-\gamma}=\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}Z_{t+1}^{\frac{1}{\psi}-\gamma}, \quad (4)$$

where

$$Z_{t+1}=\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}=\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}R_{W,t+1}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \quad (5)$$

and where the second equality in (5) implies an equivalent representation of the stochastic discount factor (4) derived by Epstein and Zin (1989), based on consumption growth and the simple gross return $R_{W,t+1}$ to a claim on aggregate consumption. We refer to this return as the market return, which in general is unobservable. If $\gamma=1/\psi$, equation (4) corresponds to the stochastic discount factor of an investor with time-separable utility and constant relative risk aversion, where only changes in the level of consumption determines an asset premium. Otherwise, there is an additional premium to compensate for changes in the welfare valuation ratio.

Following Hansen et al. (2007) and Routledge and Zin (2010), the intertemporal marginal rate

[1] Notice that the certainty equivalent, besides being decreasing in γ, is increasing in α (for $0<\alpha\leq 1$), and decreasing in κ (for $0<\kappa\leq 1$). Thus α and κ are also contribute to effective risk aversion, but on different forms than γ.

of substitution of an investor with GDA preferences is given by:

$$M_{t,t+1} = M^*_{t,t+1}\left(\frac{1+\ell I\left(\mathcal{D}_{t+1}\right)}{1+\ell\kappa^{1-\gamma}E_t\left[I\left(\mathcal{D}_{t+1}\right)\right]}\right), \tag{6}$$

where $I(\cdot)$ is an indicator function that takes the value 1 if the condition is met and 0 otherwise, $\mathcal{D}_{t+1}$ denotes the disappointing event $Z_{t+1} < \kappa$, and

$$\ell = \frac{1}{\alpha} - 1$$

is interpretable as the degree of investor's aversion to expected downside losses.

Notice that the logarithm of $M^*_{t,t+1}$ can also be written:

$$m^*_{t,t+1} = \ln\delta - \gamma\Delta c_{t+1} - \left(\gamma - \frac{1}{\psi}\right)\Delta z_{V,t+1} \tag{7}$$

where the processes in the right-hand of equation (7) are defined by

$$\Delta c_{t+1} \equiv \ln\left(\frac{C_{t+1}}{C_t}\right) = \ln C_{t+1} - \ln C_t \quad \text{and} \quad \Delta z_{V,t+1} \equiv \ln\left(\frac{V_{t+1}}{C_{t+1}}\right) - \ln\left(\frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t}\right) \tag{8}$$

and represent respectively the change in the log consumption level, or consumption growth, and the change in the log welfare valuation ratio, or welfare valuation ratio growth. It turns out from the first equality in equation (5) that the disappointing event $Z_{t+1} < \kappa$ is equivalent to $\Delta c_{t+1} + \Delta z_{V,t+1} < \ln\kappa$. Notice that the stochastic discount factor depends directly on current consumption growth, and indirectly on future consumption growths through the welfare valuation ratio growth.

The investor is worse off if the event $\mathcal{D}_{t+1}$ prevails at time $t+1$. In addition to risk aversion, he has an aversion to this particular event if $0 < \alpha < 1$, which is disappointment aversion. The investor will be better off if current consumption is high, and if the ratio of the continuation value of his lifetime utility relative to current consumption is high as well. In this form, disappointing economic conditions correspond to periods where the sum of growth rates of consumption and welfare valuation ratio is less than a specific threshold, $\ln\kappa$. In particular, if $\kappa = 1$, disappointing economic conditions correspond to periods where the sum of consumption and welfare valuation ratio growth rates is negative.

2.2 Risk and Downside Risk Adjustments of Asset Returns

For every asset i in the economy, optimal consumption and portfolio choice by the representative investor induces a restriction on its simple gross return, $R_{i,t+1}$, that is implied by the Euler condition:

$$E_t \left[M_{t,t+1} R_{i,t+1} \right] = 1. \tag{9}$$

Let $R_{f,t+1}$ and $\pi_{1,t}$ denote the risk-free simple gross return and the real-world disappointment probability, respectively defined by:

$$R_{f,t+1} = \frac{1}{E_t \left[M_{t,t+1} \right]} \quad \text{and} \quad \pi_{1,t} = E_t \left[I \left(\mathcal{D}_{t+1} \right) \right]. \tag{10}$$

Alternatively, the Euler equation (9) implies that:

$$E_t^{\mathbb{Q}} \left[R_{i,t+1}^e \right] = 0, \tag{11}$$

where $R_{i,t+1}^e = R_{i,t+1} - R_{f,t+1}$ denotes the excess return of asset i over the risk-free return, and where $E_t^{\mathbb{Q}} [\cdot]$ denotes the conditional expectation operator associated with the effective risk-adjusted density $Q_{t,t+1}$ defined by:

$$Q_{t,t+1} = \frac{M_{t,t+1}}{E_t \left[M_{t,t+1} \right]}. \tag{12}$$

Effective risk stands for the combination of both risk and disappointment, where risk represents regular covariances with consumption and welfare valuation ratio growths as usually understood, and disappointment represents expected losses conditional upon the disappointing event.

Proposition 2.1 *The Euler condition (9) can be re-written in one of the following forms:*

$$\mu_{i,t}^{\mathbb{H}} = \ell \pi_{1,t}^{\mathbb{H}} \mu_{1i,t}^{\mathbb{D}}, \qquad \mu_{i,t}^{\mathbb{H}} = (1-\alpha)\, \pi_{2,t}^{\mathbb{H}} \mu_{2i,t}^{\mathbb{U}}, \qquad \pi_{1,t}^{\mathbb{H}} \mu_{1i,t}^{\mathbb{D}} = \alpha \pi_{2,t}^{\mathbb{H}} \mu_{2i,t}^{\mathbb{U}}, \tag{13}$$

where the quantities $\mu_{i,t}^{\mathbb{H}}$, $\mu_{1i,t}^{\mathbb{D}}$ and $\mu_{2i,t}^{\mathbb{U}}$ are the expected excess return, the downside expected downside loss and the upside potential respectively, after risk corrections, evaluated under different economically meaningful probability densities. They are defined by:

$$\mu_{i,t}^{\mathbb{H}} \equiv E_t^{\mathbb{H}}\left[R_{i,t+1}^e\right], \qquad \mu_{1i,t}^{\mathbb{D}} \equiv E_t^{\mathbb{D}}\left[-R_{i,t+1}^e \mid \mathcal{D}_{t+1}\right], \qquad \mu_{2i,t}^{\mathbb{U}} \equiv E_t^{\mathbb{U}}\left[R_{i,t+1}^e \mid \mathcal{S}_{t+1}\right], \tag{14}$$

where $\mathcal{S}_{t+1}$ denotes the satisfying event $Z_{t+1} \geq \kappa$ or equivalently $\Delta c_{t+1} + \Delta z_{V,t+1} \geq \ln \kappa$, and where $E_t^{\mathbb{H}}[\cdot]$, $E_t^{\mathbb{D}}[\cdot \mid \mathcal{D}_{t+1}]$ and $E_t^{\mathbb{U}}[\cdot \mid \mathcal{S}_{t+1}]$ are respectively the conditional expectation operators associated with the risk-adjusted density $H_{t,t+1}^*$, the risk-adjusted downside density $D_{t,t+1}^*$ and the risk-adjusted upside density $U_{t,t+1}^*$, defined by:[2]

$$H_{t,t+1}^* \equiv \frac{M_{t,t+1}^*}{E_t\left[M_{t,t+1}^*\right]}, \qquad D_{t,t+1}^* \equiv \frac{M_{t,t+1}^*}{E_t\left[M_{t,t+1}^* \mid \mathcal{D}_{t+1}\right]}, \qquad U_{t,t+1}^* \equiv \frac{M_{t,t+1}^*}{E_t\left[M_{t,t+1}^* \mid \mathcal{S}_{t+1}\right]}. \tag{15}$$

The remaining quantities in (13), $\pi_{1,t}^{\mathbb{H}}$ and $\pi_{2,t}^{\mathbb{H}}$, are respectively the risk-adjusted disappointment and satisfaction probabilities defined by:

$$\pi_{1,t}^{\mathbb{H}} \equiv E_t^{\mathbb{H}}\left[I\left(\mathcal{D}_{t+1}\right)\right] \quad \text{and} \quad \pi_{2,t}^{\mathbb{H}} \equiv E_t^{\mathbb{H}}\left[I\left(\mathcal{S}_{t+1}\right)\right] = 1 - \pi_{1,t}^{\mathbb{H}}. \tag{16}$$

The proof of Proposition 2.1 can be found in the appendix.

If the investor is disappointment neutral, $\alpha = 1$, then $H_{t,t+1}^*$ coincides with the effective risk-neutral distribution $Q_{t,t+1}$. Indeed, we have from the first equation in (13) that $\mu_{i,t}^{\mathbb{H}} = 0$ if $\alpha = 1$. Notice that $\mu_{1i,t}^{\mathbb{D}}$ is the expected downside loss, adjusted for downside risk. The first equation in (13) shows that the risk-adjusted expected excess return is proportional to the risk-adjusted disappointment probability times the downside risk-adjusted expected downside loss, where the coefficient of proportionality is the degree of loss aversion ℓ. Disappointment averse investors dislike assets with $\mu_{1i,t}^{\mathbb{D}} > 0$ since they might face large losses in these assets when disappointment

[2] Since we have $E_t\left[H_{t,t+1}^*\right] = 1$, $E_t\left[D_{t,t+1}^* \mid \mathcal{D}_{t+1}\right] = 1$ and $E_t\left[U_{t,t+1}^* \mid \mathcal{S}_{t+1}\right] = 1$, then $H_{t,t+1}^*$ can be thought of as adjusting the overall distribution, $D_{t,t+1}^*$ as adjusting the downside distribution (or the distribution conditional upon disappointment), and $U_{t,t+1}^*$ as adjusting the upside distribution (or the distribution conditional upon satisfaction).

sets in, and it requires an additional premium to get them holding these assets. This in turn translates into a positive $\mu_{i,t}^{\mathbb{H}}$.

We have shown that if the investor is disappointment averse, then the risk-adjusted expected excess return $\mu_{i,t}^{\mathbb{H}}$ is not necessarily equal to zero, and it will not be in general. This is because simple risk corrections to asset prices, which are driven by covariances of their payoffs with the marginal utility of wealth, are not enough to compensate an investor who is particularly sensitive to downside losses. The higher the disappointment aversion (the lower α or equivalently the higher the degree of loss aversion ℓ), the higher the relative compensation for expected downside losses.

We also notice that $\mu_{2i,t}^{\mathbb{U}}$ is the upside potential, adjusted for upside risk. The second equation in (13), which we recall is equivalent to the first, shows that the risk-adjusted expected excess return is proportional to the risk-adjusted satisfaction probability times the upside risk-adjusted upside potential, where the coefficient of proportionality is one minus the coefficient of disappointment aversion. Most importantly, this equation shows that investors require an additional premium ($\mu_{i,t}^{\mathbb{H}} > 0$) for holding assets with $\mu_{2i,t}^{\mathbb{U}} > 0$. At a first glance, it might be hard to understand why investors may require an additional premium to invest in an asset with good upside potential.

To understand this, we now refer to the third formulation of the Euler condition, the third equation in (13). This equation shows that there is a no-arbitrage condition that relates expected downside losses to the upside potential. It shows that the risk-adjusted disappointment probability times the downside risk-adjusted expected downside loss is proportional to the risk-adjusted satisfaction probability times the upside risk-adjusted upside potential, where the coefficient of proportionality coincides with the coefficient of disappointment aversion. So, in equilibrium, assets with good upside potential are exactly those with large expected downside losses. Where there is an upside potential, there are always expected downside losses. Upside potential and expected downside losses are just the opposite sides of the same coin.

2.3 Cross-Sectional Implications of GDA Preferences

2.3.1 Substituting out Consumption

It is also important to notice from the second equality in equation (5) that the log market return is related to consumption growth and to the welfare valuation ratio growth through

$$r_{W,t+1} = -\ln\delta + \Delta c_{t+1} + \left(1 - \frac{1}{\psi}\right)\Delta z_{V,t+1}. \tag{17}$$

In this case, equation (7) becomes

$$m^*_{t,t+1} = (1-\gamma)\ln\delta - \gamma r_{W,t+1} - \left(\frac{\gamma - 1}{\psi}\right)\Delta z_{V,t+1}, \tag{18}$$

and the disappointing event $Z_{t+1} < \kappa$ is equivalent to $r_{W,t+1} + (1/\psi)\,\Delta z_{V,t+1} < \ln(\kappa/\delta)$.

This latter equation reveals that, if the elasticity of intertemporal substitution is infinite, then the logarithm of $M^*_{t,t+1}$ reduces to $m^*_{t,t+1} = (1-\gamma)\ln\delta - \gamma r_{W,t+1}$, and the disappointing event $\mathcal{D}_{t+1}$ is just equivalent to the market log return falling below a fixed specific threshold given by investor's preferences, $r_{W,t+1} < \ln(\kappa/\delta)$. So, for a representative investor who perfectly substitutes out consumption through time, the market return is the only factor determining both systematic risk and disappointment.

As we pointed out earlier, the market return $r_{W,t}$ is not directly observed by the econometrician. The return to a stock market index is sometimes used to proxy for this return as in Epstein and Zin (1991). Also, the welfare valuation ratios $z_{V,t} \equiv \ln(V_t/C_t)$ and $z_{\mathcal{R},t} \equiv \ln(\mathcal{R}_t(V_{t+1})/C_t)$ are unobservable. Following Hansen et al. (2008) and Bonomo et al. (2011), we can exploit the dynamics of aggregate consumption growth and the recursion (2) in addition to the definition of the certainty equivalent (3) to solve for the unobserved welfare valuation ratios.

From equation (17), it follows that stochastic volatility of aggregate consumption growth is a sufficient condition for stochastic volatility of the market return. In all what follows, this additional assumption is coupled with our assumption on investors' preferences. More specifically, assume for example that the logarithm of consumption follows a heteroscedastic random walk as in Bonomo et al. (2011), were the stochastic volatility of consumption growth is an AR(1) process that can be well-approximated in population by a two-state Markov chain as shown in Garcia et al. (2008). Then, it can be shown that the welfare valuation ratios satisfy

$$z_{V,t} = \varphi_{V0} + \varphi_{V\sigma}\sigma^2_{W,t} \quad \text{and} \quad z_{\mathcal{R},t} = \varphi_{\mathcal{R}0} + \varphi_{\mathcal{R}\sigma}\sigma^2_{W,t} \tag{19}$$

were $\sigma^2_{W,t} \equiv Var_t\left[r_{W,t+1}\right]$ is the conditional variance of the market return, and where the drift coefficients φ_{V0} and $\varphi_{\mathcal{R}0}$ and the loadings $\varphi_{V\sigma}$ and $\varphi_{\mathcal{R}\sigma}$ depend on investor's preference parameters and on parameters of the dynamics of consumption volatility. In this case, equation (18) becomes

$$m^*_{t,t+1} = (1-\gamma)\ln\delta^* - \gamma r_{W,t+1} - \left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\Delta\sigma^2_{W,t+1}, \tag{20}$$

and the disappointing event $Z_{t+1} < \kappa$ is equivalent to $r_{W,t+1} + (1/\psi)\,\varphi_{V\sigma}\Delta\sigma^2_{W,t+1} < \ln(\kappa/\delta^*)$, where $\Delta\sigma^2_{W,t+1} \equiv \sigma^2_{W,t+1} - \varphi\sigma^2_{W,t}$ and where

$$\ln\delta^* = \ln\delta + \frac{1}{\psi}\left(\varphi_{V0} - \varphi_{\mathcal{R}0}\right) \quad \text{and} \quad \varphi = \frac{\varphi_{\mathcal{R}\sigma}}{\varphi_{V\sigma}}.$$

Our definitions and notations for $\Delta z_{V,t+1}$ and $\Delta\sigma^2_{W,t+1}$ presume that $z_{\mathcal{R},t} \approx z_{V,t}$ and $\varphi \approx 1$. We later illustrate in a calibration exercise that this indeed is the case. This shows that changes in the welfare valuation ratio can empirically be proxied by changes in stock market volatility, where volatility can be estimated using a generalized autoregressive conditional heteroscedasticity (GARCH) model, can be computed from high-frequency index returns (realized volatility), or can be measured by the option implied volatility (VIX).

It should finally be noted that the loading coefficient $\varphi_{V\sigma}$ of the welfare valuation ratio onto the market volatility must be negative to be consistent with the empirical evidence reported by Bansal et al. (2005) that asset markets dislike macroeconomic uncertainty, and also to corroborate the theoretical predictions of long-run risks models featuring a time-varying consumption volatility process. In all what follows, we take as given that $\varphi_{V\sigma} < 0$ and will show later in the calibration assessment that this important theoretical implication of the model is met.

2.3.2 Cross-Sectional Representation of Expected Returns

Since the three equations in (13) are all equivalent, analyzing the cross-section of asset returns requires only one of them, and we will then focus on the first equation from now on. In order to derive the implied cross-sectional model in a linear beta form as common in the cross-sectional asset pricing literature, we consider the following approximations:

$$\begin{aligned} H^*_{t,t+1} &\approx 1 + \theta^*_t \left(m^*_{t,t+1} - E_t \left[m^*_{t,t+1} \right] \right) \\ D^*_{t,t+1} &\approx 1 + \theta^*_{1,t} \left(m^*_{t,t+1} - E_t \left[m^*_{t,t+1} \mid \mathcal{D}_{t+1} \right] \right) \end{aligned} \tag{21}$$

where the coefficients θ^*_t and $\theta^*_{1,t}$ are positive and ensure that the volatility and the downside volatility of $M^*_{t,t+1}$ remain unchanged under the first and the second approximations respectively.

Notice that the risk-adjusted expected excess returns $\mu^{\mathbb{H}}_{i,t}$ may be written as follows:

$$\begin{aligned} \mu^{\mathbb{H}}_{i,t} &= \mu_{i,t} + Cov_t \left(H^*_{t,t+1}, R^e_{i,t+1} \right) \\ &= \mu_{i,t} - \gamma \theta^*_t \sigma_{iW,t} - \left(\frac{\gamma - 1}{\psi} \right) \varphi_{V\sigma} \theta^*_t \sigma_{iX,t} \end{aligned} \tag{22}$$

where $\mu_{i,t} \equiv E_t \left[R^e_{i,t+1} \right]$ is the expected excess return, while $\sigma_{iW,t} \equiv Cov_t \left(R^e_{i,t+1}, r_{W,t+1} \right)$ denotes the covariance between excess returns and the market return, and $\sigma_{iX,t} \equiv Cov_t \left(R^e_{i,t+1}, \Delta\sigma^2_{W,t+1} \right)$ denotes the covariance between excess returns and changes in market volatility.

We show that the risk-adjusted expected downside loss $\mu^{\mathbb{D}}_{1i,t}$ may be written

$$\begin{aligned} \mu^{\mathbb{D}}_{1i,t} = -\mu_{i,t} - \frac{1}{\pi_{1,t}} \Bigg[& \left(1 + \gamma \theta^*_{1,t} \mu_{1W,t} + \left(\frac{\gamma - 1}{\psi} \right) \varphi_{V\sigma} \theta^*_{1,t} \mu_{1X,t} \right) \sigma_{i\mathcal{D},t} \\ & - \gamma \theta^*_{1,t} \sigma_{iW\mathcal{D},t} - \left(\frac{\gamma - 1}{\psi} \right) \varphi_{V\sigma} \theta^*_{1,t} \sigma_{iX\mathcal{D},t} \Bigg] \end{aligned} \tag{23}$$

where $\sigma_{iW\mathcal{D},t} \equiv Cov_t \left(R^e_{i,t+1}, r_{W,t+1} I \left(\mathcal{D}_{t+1} \right) \right)$ and $\sigma_{iX\mathcal{D},t} \equiv Cov_t \left(R^e_{i,t+1}, \Delta\sigma^2_{W,t+1} I \left(\mathcal{D}_{t+1} \right) \right)$ and $\sigma_{i\mathcal{D},t} \equiv Cov_t \left(R^e_{i,t+1}, I \left(\mathcal{D}_{t+1} \right) \right)$ denote covariances between excess returns and three outcomes that are contingent to the disappointing event, and where the quantities $\mu_{1W,t} \equiv E_t \left[r_{W,t+1} \mid \mathcal{D}_{t+1} \right]$ and $\mu_{1X,t} \equiv E_t \left[\Delta\sigma^2_{W,t+1} \mid \mathcal{D}_{t+1} \right]$ represent the downside means of the market return and changes in market volatility, respectively.

Substituting (22) and (23) into the first equation in (13) and solving for expected excess returns, we show that the cross-sectional risk-return tradeoff may be written in linear covariance form as

$$\mu_{i,t} \approx p_{W,t}\sigma_{iW,t} + p_{X,t}\sigma_{iX,t} + p_{\mathcal{D},t}\sigma_{i\mathcal{D},t} + p_{W\mathcal{D},t}\sigma_{iW\mathcal{D},t} + p_{X\mathcal{D},t}\sigma_{iX\mathcal{D},t} = p_{F,t}^{\top}\sigma_{iF,t} \tag{24}$$

where the corresponding risk prices are given by

$$\begin{aligned}
p_{W,t} &= \frac{\theta_t^*}{1+\ell\pi_{1,t}\left(1+\eta_t\right)}\gamma \quad \text{and} \quad p_{X,t} = \frac{\theta_t^*}{1+\ell\pi_{1,t}\left(1+\eta_t\right)}\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}, \\
p_{\mathcal{D},t} &= -\frac{\ell\left(1+\eta_t\right)}{1+\ell\pi_{1,t}\left(1+\eta_t\right)}\left(1+\gamma\theta_{1,t}^*\mu_{1W,t}+\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\theta_{1,t}^*\mu_{1X,t}\right), \\
p_{W\mathcal{D},t} &= \frac{\ell\left(1+\eta_t\right)\theta_{1,t}^*}{1+\ell\pi_{1,t}\left(1+\eta_t\right)}\gamma \quad \text{and} \quad p_{X\mathcal{D},t} = \frac{\ell\left(1+\eta_t\right)\theta_{1,t}^*}{1+\ell\pi_{1,t}\left(1+\eta_t\right)}\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}.
\end{aligned} \tag{25}$$

The quantity η_t represents the relative disappointment probability spread given by

$$\eta_t \equiv \frac{\pi_{1,t}^{\mathbb{H}}}{\pi_{1,t}} - 1 \approx \gamma\theta_t^*\left(\mu_{W,t}-\mu_{1W,t}\right) + \left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\theta_t^*\left(\mu_{X,t}-\mu_{1X,t}\right), \tag{26}$$

where $\mu_{W,t} \equiv E_t\left[r_{W,t+1}\right]$ and $\mu_{X,t} \equiv E_t\left[\Delta\sigma_{W,t+1}^2\right]$ are the means of the market return and changes in market volatility, respectively.

Equation (24) corresponds to a linear multifactor model representation of expected excess returns. In the unrestricted case, we have a five-factor model. In addition to the market return and changes in market volatility which are shown to be cross-sectional pricing factors in Ang, Hodrick, Xing and Zhang (2006) and Adrian and Rosenberg (2008), three additional factors command a risk premium. These factors are all payoffs which are contingent to the disappointing event, making them interpretable as options. Recalling that $\varphi_{V\sigma} < 0$, the disappointing event, $r_{W,t+1} + (1/\psi)\,\varphi_{V\sigma}\Delta\sigma_{W,t+1}^2 < \ln\left(\kappa/\delta^*\right)$, may occur due to a fall in the market return or an increase in changes in market volatility, or both. This means that the three options mature in-the-money if the market return falls or if changes in market volatility increase. For this reason, depending on the nature of option payoff, they can be seen as either put options on the market return or call options on changes in market volatility.

More specifically, the factor $I\left(\mathcal{D}_{t+1}\right)$ is a binary cash-or-nothing option. It is both interpretable as either a binary cash-or-nothing put on the market return or a binary cash-or-nothing call on changes in market volatility. The factor $r_{W,t+1}I\left(\mathcal{D}_{t+1}\right)$ would then be a put option on the market return, since the option payoff depends on the market return. Similarly, the factor $\Delta\sigma^2_{W,t+1}I\left(\mathcal{D}_{t+1}\right)$ would be seen as a call option on changes in market volatility, since the contingent payoff depends on changes in market volatility. We further characterize these options in some special cases.

Consider the restricted case where $\psi = \infty$. We have already shown that the downside event reduces to $r_{W,t+1} < \ln\left(\kappa/\delta\right)$, and now the relative disappointment probability spread also reduces to $\eta_{W,t} = \gamma\theta_t^*\left(\mu_{W,t} - \mu_{1W,t}\right)$, where $\mu_{1W,t} = E_t\left[r_{W,t+1} \mid r_{W,t+1} < \ln\left(\kappa/\delta\right)\right]$. The restriction $\psi = \infty$ implies that $p_{X,t} = p_{X\mathcal{D},t} = 0$. Thus, changes in market volatility and the call option are not priced. The cross-sectional model then reduces to a three-factor model with the market return, the binary cash-or-nothing option and the put option. The associated risk prices are given by

$$\begin{aligned} p_{W,t} &= \frac{\theta_t^*}{1+\ell\pi_{1,t}\left(1+\eta_{W,t}\right)}\gamma, \\ p_{\mathcal{D},t} &= -\frac{\ell\left(1+\eta_{W,t}\right)}{1+\ell\pi_{1,t}\left(1+\eta_{W,t}\right)}\left(1+\gamma\theta_{1,t}^*\mu_{1W,t}\right) \quad \text{and} \quad p_{W\mathcal{D},t} = \frac{\ell\left(1+\eta_{W,t}\right)\theta_{1,t}^*}{1+\ell\pi_{1,t}\left(1+\eta_{W,t}\right)}\gamma. \end{aligned} \tag{27}$$

To further illustrate our interpretation of the new factors in the special case $\psi = \infty$, assume for example that $\kappa = \delta$. Then, the disappointment event becomes $r_{W,t+1} < 0$ and we have

$$I\left(\mathcal{D}_{t+1}\right) = I\left(W_{t+1} < W_t\right) \quad \text{and} \quad -r_{W,t+1}I\left(\mathcal{D}_{t+1}\right) = \max\left(w_t - w_{t+1}, 0\right) \approx \frac{\max\left(W_t - W_{t+1}, 0\right)}{W_t},$$

where W_t denotes the aggregate wealth and $w_t = \ln W_t$. Clearly, this shows that our two option factors represent a regular binary cash-or-nothing put option and a conventional European put option on aggregate wealth, with a maturity of one period and a strike equal to current wealth.

It is important to determine what characteristic of investors' behavior is responsible for a command of a premium related to a specific factor at the market place. Equations (25) and (27) reveal that $p_{W,t} \neq 0$ if and only if $\gamma \neq 0$, regardless of the disappointment aversion parameter ℓ. This shows that compensation for the covariance with the market return is exclusively due to investors' risk aversion. The asset pricing literature generally agrees on investors' risk aversion parameter $\gamma > 1$. Taking this as given, it then follows from equation (25) that $p_{X,t} \neq 0$ if and only if $\psi \neq \infty$, regardless of the disappointment aversion parameter ℓ. Thus, we can argue that compensation for the covariance with changes in market volatility is exclusively due to imperfect intertemporal substitution of consumption. Investor's risk aversion ($\gamma > 1$) and imperfect intertemporal substitution of consumption ($\psi < \infty$) both imply that $p_{W,t} > 0$ and $p_{X,t} < 0$. Thus, consistent with the existing theoretical and empirical literature (see for example Ang, Hodrick, Xing and Zhang 2006; Adrian and Rosenberg 2008), investors require a premium for a security that has a low return when the market return is low ($\sigma_{iW,t} > 0$), but are willing to pay a premium for a security that pays off when changes in market volatility are high ($\sigma_{iX,t} > 0$).

On the other hand, equations (25) and (27) also reveal that $p_{\mathcal{D},t} \neq 0$ if and only if $\ell \neq 0$, regardless of other preference parameters. This shows that compensation for the covariance with the cash-or-nothing option is exclusively due to disappointment aversion. This model-implied premium when $\ell > 0$ stems from the preference of investors for securities with high returns when the disappointing event occurs ($\sigma_{i\mathcal{D},t} > 0$).

We also observe that, $p_{W\mathcal{D},t} \neq 0$ if and only if both $\gamma \neq 0$ and $\ell \neq 0$. This shows that neither risk aversion alone, nor disappointment aversion alone suffices to explain the requirement for investors to be compensated for the covariance with the put option on the market return. Similarly, presuming that $\gamma > 1$, then $p_{X\mathcal{D},t} \neq 0$ if and only if both $\psi \neq \infty$ and $\ell \neq 0$. It turns out that neither imperfect intertemporal substitution of consumption alone, nor disappointment aversion alone suffices to explain the requirement for investors to be compensated for the covariance with the call option on changes in market volatility. Investor's risk aversion ($\gamma > 1$), imperfect intertemporal substitution of consumption ($\psi < \infty$) and disappointment aversion ($\ell > 0$) all imply that $p_{W\mathcal{D},t} > 0$ and $p_{X\mathcal{D},t} < 0$. This shows that investors require a premium for a security that has a low return when a low market return in a disappointing state further decreases ($\sigma_{iW\mathcal{D},t} > 0$), and are willing to pay a premium for a security that pays off when large changes in market volatility in a disappointing state further increase ($\sigma_{iX\mathcal{D},t} > 0$).

The cross-sectional risk-return relation (24) is finally equivalent to:

$$\mu_{i,t} \approx \lambda_{F,t}^{\top} \beta_{iF,t} \tag{28}$$

where $\beta_{iF,t}$ is the vector containing the multivariate regression coefficients of asset excess returns onto the factors, and $\lambda_{F,t}$ is the vector of factor risk premiums, respectively given by

$$\beta_{iF,t} = \Sigma_{F,t}^{-1}\sigma_{iF,t} \quad \text{and} \quad \lambda_{F,t} = \Sigma_{F,t}p_{F,t} \tag{29}$$

where $\sigma_{iF,t}$ is the vector of covariances of the asset excess return with the priced factors, and where $\Sigma_{F,t}$ is the factor covariance matrix. It is important to note that if the covariance between the daily return on the market and daily changes in the volatility of the market $(Cov(r_{W,t}, \Delta\sigma^2_{W,t}))$ is negative or sufficiently close to zero[3], than the signs of the elements of $\lambda_{F,t}$ are the same as of the corresponding elements of $p_{F,t}$. This beta representation nests both the five-factor case $(\psi \neq \infty)$ and the three-factor case $(\psi = \infty)$. An extensive empirical investigation of these betas for the cross-section of stock returns will be carried out in subsequent sections.

3 Calibration Assessment

In this section, we calibrate an endowment economy and discuss the major quantities derived analytically in previous sections. The focus will be on the factor risk premia λ_F implied by the model, which we will further compare to the values estimated in an extensive cross-sectional study using actual data in Section 4.3. We follow Bonomo et al. (2011) in modeling and calibrating the endowment process, and solving for asset prices in closed form. We assume that consumption growth is unpredictable and that its conditional variance fluctuates according to a Markov variable s_t, which can take a different value in each of the N states of nature of the economy. The sequence s_t evolves according to a transition probability matrix P defined as:

$$P^\top = [p_{ij}]_{1\leq i,j\leq N}\,, \quad p_{ij} = Prob\,(s_{t+1} = j \mid s_t = i)\,. \tag{30}$$

As in Hamilton (1994), let $\zeta_t = e_{s_t}$, where e_j is the $N \times 1$ vector with all components equal to zero but the jth component equals one.

[3] The later is supported in our data.

Formally, consumption growth is modeled as follows:

$$\Delta c_{t+1} = \mu_c + \sigma_t \varepsilon_{c,t+1} \tag{31}$$

where $\varepsilon_{c,t+1} \mid \langle \varepsilon_{c,\tau}, \tau \leq t; \zeta_m, m \in \mathbb{Z} \rangle \sim \mathcal{N}(0,1)$ and where $\sigma_t = \sqrt{\omega_c^\top \zeta_t}$ is the volatility of consumption growth. The scalar μ_c is the expected consumption growth, and the vector ω_c contains consumption volatility in each state of nature, where the component j of a vector refers to the value in state $s_t = j$. Given these endowment dynamics, we solve for welfare valuation ratios in closed form, which we combine to consumption growth to derive the endogenous market return and variance processes.

To calibrate the model, we assume two states for the Markov chain so that consumption conditional variance σ_t^2 behaves like an AR(1) process with mean μ_σ, persistence ϕ_σ, volatility σ_σ, positive skewness and zero excess kurtosis. The two states of the economy naturally corresponds to a low (L) and a high (H) volatility states. We calibrate the consumption process at the monthly decision interval to match actual sample mean and volatility of real annual US consumption growth from 1930 to 2010.

The mean of consumption growth is calibrated to $\mu_c = 0.15 \times 10^{-2}$ and its volatility, which is equal to $\sqrt{\mu_\sigma}$, is calibrated to $\sqrt{\mu_\sigma} = 0.7305 \times 10^{-2}$. The volatility of consumption volatility is calibrated to $\sigma_\sigma = 0.6263 \times 10^{-4}$ and we set the persistence to $\phi_\sigma = 0.995$ in our benchmark case. We will further study the sensitivity of the quantities when we vary the persistence ϕ_σ as well as preference parameters. In our benchmark case, the implied state values of expected consumption growth are $\mu_c(L) = \mu_c(H) = 0.15\%$. The state values of consumption volatility are $\sigma(L) = 0.46\%$ and $\sigma(H) = 1.32\%$. The state transition probabilities are $p_{LL} = 0.9989$ and $p_{HH} = 0.9961$, and the corresponding long-run probabilities are $\pi_L = 0.7887$ and $\pi_H = 0.2113$.

We set the value of the risk aversion parameter γ to 3.75 and the elasticity of intertemporal substitution ψ to 1.5, and we consider several scenarios were we vary the values of the other preference parameters. In our benchmark scenario, we consider $\delta = 0.9979$, $\alpha = 0.3$ and $\kappa = 0.992$. All the scenarios are shown in Table 1. The model-implied annualized (time-averaged) mean, volatility and first-order autocorrelation of consumption growth are respectively 1.80%, 2.21% and 0.25%, and are consistent with the observed annual values of 1.88%, 2.21% and 0.46%, respectively.

In Panel A, we observe that across all scenarios, the annualized (time-averaged) mean risk-free rate varies between 0.74% and 1.97%, and the corresponding volatility between 1.69% and 4.78%. These scenarios' values are consistent with the estimated risk-free rate mean of 1.21% and volatility 4.10%. In our base case, the values are 1.31% and 2.46%. Panel B shows that the welfare valuation ratio loads negatively on market volatility, consistent with the economic intuition that asset values and consequently investor's wealth and welfare fall in periods of high uncertainty in financial markets. Panel C shows that the disappointment probability is higher in periods of high volatility versus low volatility periods. Also, increasing the parameter κ increases the number of disappointing outcomes and consequently the disappointment probability. In our base case, the disappointing event has a probability of 1.22% in the low volatility state, 21.05% in the high volatility state and 5.41% in the long run.

Panel D shows monthly model-implied factor risk premia. These values will be confronted to their data counterparts estimated in the next empirical section. The market factor risk premium λ_W is larger in the high volatility state and smaller in the low volatility state. The expected market risk premium ranges between 0.0051 and 0.0075. Similarly, the factor premium associated to the cash-or-nothing option $\lambda_{\mathcal{D}}$ is larger in the high volatility state versus the low volatility states. Its expected value ranges from -0.2560 to -0.1175. This premium is negative as the relative downside potential of a risky asset is negative, leading to a positive compensation. To the contrary, the volatility factor risk premium λ_X is larger in the low volatility state versus the high volatility state, and its expected value ranges between -1.16E-5 to -6.29E-6. The volatility factor risk premium is also negative as a risky asset with negative volatility beta commands a positive volatility premium. The other factor risk premia are associated to the put option on the market return ($\lambda_{W\mathcal{D}}$) and the call option on changes in market volatility ($\lambda_{X\mathcal{D}}$). Their expected values ranges between 0.0031 and 0.0061, and between -1.12E-5 and -5.23E-6. In the next section, we empirically estimate these average risk premia using observed individual stock returns and compare the estimated values to the model-implied values.

4 Empirical Assessment

The cross-sectional risk-return relation (24) is in the centre of our empirical assessment:

$$\mu_{i,t} \approx p_{W,t}\sigma_{iW,t} + p_{X,t}\sigma_{iX,t} + p_{\mathcal{D},t}\sigma_{i\mathcal{D},t} + p_{W\mathcal{D},t}\sigma_{iW\mathcal{D},t} + p_{X\mathcal{D},t}\sigma_{iX\mathcal{D},t} \tag{32}$$

where $\sigma_{if,t}$ denotes the covariance between the excess return of asset i and factor f, while $p_{f,t}$ is the price of risk given in (25) for different factors. In the general case there are five priced factors given by

$$F_t^\top = \begin{pmatrix} r_{W,t} & \Delta\sigma_{W,t}^2 & I(\mathcal{D}_t) & r_{W,t}I(\mathcal{D}_t) & \Delta\sigma_{W,t}^2 I(\mathcal{D}_t) \end{pmatrix}. \tag{33}$$

We refer to this as the **GDA5** (generalized disappointment aversion) model throughout the rest of the paper. In the restricted case $\psi = +\infty$, the number of risk factors reduces to three (the **GDA3** model):

$$F_t^\top = \begin{pmatrix} r_{W,t} & I(\mathcal{D}_t) & r_{W,t}I(\mathcal{D}_t) \end{pmatrix}. \tag{34}$$

So far, our definition of the disappointing event has been very general (defined in terms of the parameters of the model). However, we have to be more specific to be able to carry out the empirical analysis. We have decided to define the disappointing event as simply as possible: $\mathcal{D}_t$ corresponds to $r_{W,t} < 0$, i.e. when the log market return falls below zero. In the case of the GDA3 model, the downside event is $r_{W,t} < \ln(\kappa/\delta)$, so our definition used in the empirical analysis is equal to assuming $\kappa = \delta$. In the more general case of the GDA5 model, the disappointing region is defined as $r_{W,t} + (1/\psi)\,\varphi_{V\sigma}\Delta\sigma_{W,t}^2 < \ln(\kappa/\delta^*)$. So, assuming $r_{W,t} < 0$ means that we disregard the second term from the left hand side of the inequality, and assume $\kappa = \delta^*$. We have to emphasize that our results do not hinge on this particular definition of the disappointing event. In the section of robustness checks (4.3.3) we investigate how the results change if disappointment regions of the form[4]

$$r_{W,t} - a\,\frac{\sigma_{r_W}}{\sigma_{\Delta\sigma_W^2}}\Delta\sigma_{W,t}^2 < b \tag{35}$$

[4]This form corresponds closely to the definition of the downside event in the case of the more general GDA5 model. We consider different values of the parameters a and b.

are considered. We conclude that our results are very robust to changes in the definition of $\mathcal{D}_t$.

4.1 Data

Following common practice in the literature, we test our model using all common stocks (CRSP share codes 10 and 11) traded on the NYSE, AMEX and NASDAQ markets. The source of the data is the Center for Research in Security Prices (CRSP). The analysis covers the period between July, 1963 and December, 2010.

The market return is the value-weighted average return on all NYSE, AMEX, and NASDAQ stocks from CRSP, while the risk free rate is the one-month US Treasury bill rate from Ibbotson Associates. Both time series are obtained from Kenneth R. French's data library[5]. To follow the implications of our theoretical model as closely as possible, we use the log market return (not excess return) in our empirical tests. However, it is worth noting that the results are basically unchanged when the simple market excess return is used instead.

In order to be able to test the GDA5 model, we need to measure changes in the volatility of the market return. Several approaches have been used for measuring market volatility in studies analyzing the cross-section of stock returns: Ang, Hodrick, Xing and Zhang (2006) use the VIX index, Adrian and Rosenberg (2008) estimate a model with conditional heteroskedasticity, while Bandi et al. (2006) use realized volatility. We have chosen to use the model based approach in our main specification. The most important advantage of this approach is that it lets us use the entire sample period[6]. We obtain our measure of market volatility by fitting an EGRACH model (introduced by Nelson; 1991) to the daily market return series using the whole sample period. The exact model specification and the coefficient estimates are presented in Table 2. In the section for robustness checks (4.3.3), we show how the results change if alternative measures of market volatility are considered.

When presenting our results, we will compare the performance of our model to the four factor model of Carhart (1997). Daily return series of the factors were collected from Kenneth R. French's

[5]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[6]We can obtain daily VIX and realized market volatility data starting from 1986 only.

data library.

4.2 Portfolio sorts

A lot of studies analyzing the cross-section of stock returns use portfolio sorts as their main empirical tool. They sort stocks into portfolios based on a specific measure of risk, and then examine the patterns in the average returns of these portfolios. We also start by presenting results of portfolio sorts.

We closely follow the methodology of Ang, Chen and Xing (2006): at the end of each one-year period at month t, we calculate realized covariances from (32) using daily data over the previous 12-month period. For each stock, we also calculate the average monthly excess return over the same 12-month period. Stocks are then sorted into five quintiles based on their realized covariances, and the average returns on these quintile portfolios are calculated. We repeat the same procedure for the next month, and continue throughout the whole sample period. Finally, we take the time-series average of the portfolio returns. According to Ang, Chen and Xing (2006) this use of overlapping information is more efficient, but induces moving average effects. To account for this, we report t-statistics that are adjusted using 12 Newey and West (1987) lags[7].

4.2.1 Sorting on realized covariances

The first five columns of Table 4 present average annualized returns of portfolios created by sorting stocks based on their realized covariances with our factors. The first column shows the results when $\sigma_{iW} = Cov(R^e_{i,t}, r_{W,t})$ is used. Note that this is numerically equivalent to sorting on the standard CAPM beta[8] . We can see a monotonically increasing pattern between realized average returns and realized beta for both the equal- and value-weighted portfolios. This is consistent with $p_W > 0$ indicated by theory. Also, these findings (both the pattern and the size of the spread) are in line with the literature (see for example Ang, Chen and Xing; 2006 or Ruenzi and Weigert; 2011).

[7]Note also, that the results of the portfolio sorts are essentially the same if we use non-overlapping one-year periods. The results are available upon request.

[8]Sorting on $\sigma_{iW} = Cov\left(R^e_{i,t}, r_{W,t}\right)$ is equivalent to sorting on β_{iCAPM} from the regression $R^e_{i,t} = \alpha_i + \beta_{iCAPM} \cdot r_{W,t} + \varepsilon_{it}$, since $\beta_{iCAPM} = \frac{Cov\left(R^e_{i,t}, r_{W,t}\right)}{Var(r_{W,t})}$, and the denominator does not vary in the cross-section.

In the second column, stocks are sorted into portfolios based on their covariance with the binary cash-or-nothing option, $\sigma_{i\mathcal{D}} = Cov(R^e_{i,t}, I(\mathcal{D}_t))$. As it is shown in the appendix (A.1), this is numerically equivalent to sorting on $E[R^e_{i,t} \mid \mathcal{D}_t] - E[R^e_{i,t}]$, the relative downside potential. A factor like this has not been studied in the cross-section of stock returns so far. An asset with a low $\sigma_{i\mathcal{D}}$ is undesirable because it has lower expected payoffs than usual when disappointment sets in. So, investors need compensation for holding stocks with low relative downside potential. In line with this reasoning, we can see monotonically decreasing returns when we go from low to high values of the risk measure. The difference "H-L" is significant at the 1% level for equal-weighted portfolios, and at 10% level for the value-weighted portfolios.

The third column presents portfolios when stocks are sorted on their covariance with the payoff of the put option on the market return, $\sigma_{iW\mathcal{D}} = Cov(R^e_{i,t}, r_{W,t} I(\mathcal{D}_t))$. Assets that covary positively with the put option are undesirable because they tend to have low payoffs when the market is doing bad. In line with this and the positive sign on $p_{W\mathcal{D}}$, we can see a monotonically increasing pattern across portfolios. This measure produces the largest spread in portfolio returns if we consider equal-weighted portfolios.

The fourth covariance risk measure is $\sigma_{iX} = Cov(R^e_{i,t}, \Delta\sigma^2_{W,t})$, i.e. the covariance with the change in the variance of the market. Factors similar to this have already been studied in the context of the cross-section of stock returns (e.g. Ang, Hodrick, Xing and Zhang (2006)). In line with their results and with $p_X < 0$, we find that stocks with high sensitivities to innovations in market variance have low average returns.

The fifth column shows the results if the stocks are sorted on their covariance with the payoff of the call option on changes in market volatility, $\sigma_{iX\mathcal{D}} = Cov(R^e_{i,t}, \Delta\sigma^2_{W,t} I(\mathcal{D}_t))$. So far, not much attention has been devoted to this factor in the literature. Theory indicates that $p_{X\mathcal{D}} < 0$. Accordingly, wee can see monotonically decreasing returns in Table 4, i.e. stocks with high sensitivities to innovations in market variance in bad times have low average returns. Note, that this beta measures produces the largest spread in portfolio returns if we consider value-weighted portfolios, and the "H-L" difference is highly statistically significant regardless of the weighting scheme.

The general conclusion is that all the five risk measures generate monotonic patterns in the average returns of sorted portfolios. Moreover, these patterns are in line with the signs on the prices of risk suggested by theory (25). However, there is one problem with these measures that makes it hard to disentangle their effects: they are highly correlated with each other. First, let us concentrate on the upper left corner of Table 3. The correlations between σ_{iW}, $\sigma_{i\mathcal{D}}$, and $\sigma_{iW\mathcal{D}}$ are very high, even the lowest (in magnitude) correlation is -0.86. This is in line with the literature: both Ang, Chen and Xing (2006) and Post et al. (2010) find that the regular CAPM beta and measures of downside market risk are highly correlated[9]. Also the two measures of sensitivity to changes in market variance (σ_{iX} and $\sigma_{iX\mathcal{D}}$) have a correlation of 0.68.

One way to disentangle the effect of different factors is to calculate the risk measures together, in a multivariate framework, instead of calculating them separately.

4.2.2 Sorting on multivariate betas

Instead of calculating the realized covariances with our factors separately, we run the following regression (corresponding to our GDA5 model)

$$R^e_{i,t} = \alpha_i + \beta_{iW} \cdot r_{W,t} + \beta_{iW\mathcal{D}} \cdot r_{W,t} I(\mathcal{D}_t) + \beta_{i\mathcal{D}} \cdot I(\mathcal{D}_t) + \beta_{iX} \cdot \Delta\sigma^2_{W,t} + \beta_{iX\mathcal{D}} \cdot \Delta\sigma^2_{W,t} I(\mathcal{D}_t) + \varepsilon^i_t \; , \qquad (36)$$

and use the estimated β_{if}-s in the same sorting exercise (following the same methodology) as above. Note that estimating betas from the above regression exactly corresponds to the first equation in (29). In this subsection we are focusing only on the betas calculated using all the five factors from the GDA5 model. However, we would like to note that if the first three betas (β_{iW}, $\beta_{iW\mathcal{D}}$, and $\beta_{i\mathcal{D}}$) are calculated from the GDA3 model, the results are very similar. For all the three factors, the correlation between β_{iF} calculated from the GDA3 and the GDA5 models is 0.99.

Table 3 also shows the average cross-sectional correlations between the betas calculated from the regression (36). The general message is that the correlations between these betas are much lower in magnitude than the correlations we have seen in between the σ_{if} measures. It is important to note though, that σ and β corresponding to the same factor can be very different, because now

[9]We will discuss in Section 5 the relationship between our measures and measures of downside market risk that have been used in the literature.

the measures are calculated together.

The results of the sorting exercise using these betas are presented in the last five columns of Table 4. As we have pointed out earlier, the signs of the λ-s in (28) are the same as the signs of the corresponding p-s, so we expect the same patterns that we have seen when sorting based on realized covariances. β_{iW} generates a modest spread across the portfolios, but the "H-L" difference is not significant at the 5% level. When sorting stocks into portfolios based on $\beta_{i\mathcal{D}}$, the average return on these portfolios seem to be constant. This means that $\beta_{i\mathcal{D}}$ fails to create the desired spread in the average returns. $\beta_{iW\mathcal{D}}$, on the other hand, is able to create a nice monotonically increasing pattern in the average returns. The "H-L" difference is statistically significant for both the equal- and the value-weighted portfolios. When stocks are sorted based on their sensitivity to changes in the variance of the market (β_{iX}), we can see the decreasing pattern suggested by the theory. The difference between portfolio 5 and portfolio 1 is significant at the 5% level for both weighting schemes. Lastly, $\beta_{iX\mathcal{D}}$ also delivers the decreasing pattern predicted by the theory, with highly significant "H-L" differences.

All in all, we can conclude that apart from $\beta_{i\mathcal{D}}$, all the betas create the expected patterns in the average returns of sorted portfolios. However, sorting stocks into portfolios is not the most appropriate tool in this case, since our model implies a specific relationship (the GDA3 or GDA5 model), and not one specific measure. Thus, we have to estimate the effects of several measures of risk at the same time. Based on this argument, our main tool for the empirical analysis will be Fama-MacBeth (1973) (FM) regressions.

4.3 Fama-MacBeth regressions

The starting point is the beta-form of our cross-sectional risk-return relation (28). Calculating the betas from this relationship ($\beta_{iF,t} = \Sigma_{F,t}^{-1}\sigma_{iF,t}$) is numerically equivalent to running the time-series regression (36) for each asset i in the first stage of the FM procedure. The second stage of the FM procedure (the cross-sectional regressions) corresponds to estimating the relationship $\mu_{i,t} \approx \lambda_{F,t}^{\top}\beta_{iF,t}$. As the result of the Fama-MacBeth procedure, we will obtain the average lambdas over the sample period ($E\left[\lambda_{F,t}\right]$).

We would like to take into account the conditional nature of the cross-sectional relationship in (28). We follow the spirit of Lewellen and Nagel (2006), and instead of trying to determine the appropriate set of conditioning variables, we use short-window regressions to calculate the factor loadings. At the end of each one-year period at month t, we estimate the conditional betas $\beta_{iF,t}$ using daily data from the last twelve months $(t-11,...,t)$. This approach will result again in overlapping information when calculating the conditional factor loadings. To account for this, we report Newey-West (1987) adjusted standard errors in all our tests[10].

4.3.1 Individual stocks as base assets

The majority of asset pricing studies testing expected return relations in the cross section use portfolios. However, Ang et al. (2010) have recently argued that creating portfolios destroys important information and leads to larger standard errors. They conclude their study by pointing out that using individual stocks permits more efficient tests of whether factors are priced, and there should be no reason to create portfolios. Cremers et al. (2011), Lewellen (2011) and Ruenzi and Weigert (2011) are recent examples focusing on individual stocks as base assets in Fama-MacBeth regressions. We follow this strand of the literature by considering individual stocks from the CRSP universe as base assets for the FM regressions. We consider these as our main results. However, we also present results with portfolios as base assets in Section 4.3.2.

Another decision to make is whether to use contemporaneous returns (i.e. returns over the same interval on which the conditional betas are estimated) in the cross-sectional regressions, or to use future returns (i.e. returns after the period when betas are estimated). Ang, Hodrick, Xing and Zhang (2006) argue that in order to have a factor risk explanation, there should be contemporaneous patterns between factor loadings and average returns. There are numerous studies in the asset pricing literature focusing on this contemporaneous relationship (e.g. Ang, Chen and Xing (2006), Cremers et al. (2011), Fama and MacBeth (1973), Lewellen and Nagel (2006) and Ruenzi and Weigert (2011), among others). In line with this literature, we obtain our main results using this contemporaneous approach: at every month t, we relate the conditional betas to the

[10]Note again, that the results are essentially the same if we use non-overlapping one-year periods. The results are available upon request.

average monthly excess returns over the same period on which the betas are estimated (months $t-11, ..., t$).

Results from analyzing the contemporaneous relationship between factor loadings and returns using individual stocks as base assets are presented in Table 5. Our theory implies that there should be no constant in the cross-sectional regressions. However, since there is no consensus in the empirical literature whether to include the constant or not, we report our specifications both with and without the constant term. The top panel of Table 5 presents the lambda estimates together with their statistical significance and the adjusted R^2 for the given model (where applicable). The lower panel of the table tries to give a picture about the economic significance of the results. It displays average annualized Sharpe ratios of well-diversified single exposures to the given factors. These are hypothetical portfolios that are exposed to the risk coming from only one of the factors, and are immune to the risk represented by the other factors. To have a benchmark in mind, the Sharpe ratio of the market portfolio using the same methodology is 0.557.

The first two columns of Table 5 correspond to the basic CAPM, where the only priced factor is the market return. In both cases (with and without a constant) we see a significant positive lambda on the market factor. We note also, that the constant is significant at the 10% level when included in the estimation.

Let us focus now on the results from the GDA5 model, presented in columns 5 and 6. We consider these as the main results of our empirical investigation. When the constant is included, all the five lambda estimates are significant at the 1% level, and the estimated constant is no longer significant. Regarding the signs and magnitudes of these estimates: they closely correspond to the predictions of our theoretical model. If we compare these estimates to the $E[\lambda_f]$ values from Panel D of Table 1, we see that they are surprisingly close to each other. In economic terms the two factors with the biggest effect are those that arise only if the investor is both risk averse and disappointment averse ($\lambda_{W\mathcal{D}}$ and $\lambda_{X\mathcal{D}}$). A well-diversified single exposure to the put option on the market return has an average annualized Sharpe ratio of 1.09. The Sharpe ratio of the exposure to the call option on changes in market volatility is 0.86 on average. It seems that exposure to the cash-or-nothing binary option has the lowest effect of all the five factors with a Sharpe ratio of 0.5.

When the constant term is excluded from the estimation, λ_X loses its significance and decreases in magnitude. All the other estimates seem to be robust to the exclusion of the constant.

Columns 3 and 4 of Table 5 present the results for the GDA3 model. The results are very similar to the corresponding lines of the GDA5 model. This implies that leaving out the factors connected to changes in market volatility from the model does not change the effect of the other three factors.

Columns 7 and 8 show the lambda estimates for the four factor model of Carhart (1997). The size and momentum factors are positive and significant. The economic significance of the momentum factor is particularly big. λ_{HML} is insignificant and has a negative sign. While this result seems to be puzzling at first, Ang et al. (2010) points out that when the estimation uses individual stocks, the HML premium is negative. They argue that the book-to-market effect is a characteristic effect rather than a reward for bearing HML factor loading risk. If the book-to-market ratio is included instead of the HML factor, the coefficient on B/M is strongly positive (Ang et al.; 2010). They also argue that when portfolios sorted on B/M are used as base assets in the FM procedure, the HML factor loadings are induced to have a positive coefficient through forcing the portfolio breakpoints to be based on book-to-market characteristics. This is what we also see in the results of Section 4.3.2.

The last two columns present specifications where both our five factors and the Carhart (1997) factors are included in the model. The important observation here is that the sign, magnitude, and significance of the lambda estimates do not change considerably compared to the models in columns 5 and 6. This suggests that our factors capture different channels than the factors of Carhart (1997).

To conclude this subsection, the results of Fama-MacBeth regressions analyzing the contemporaneous relationship between returns and factor loadings suggest that all the factors of the GDA5 model are priced in the cross-section of stock returns. The estimates on the prices of risk are significant both statistically and economically. Moreover, their signs and magnitudes closely correspond to the theoretical predictions.

Realized betas and future returns

While we consider the results of the previous section as our main ones, we would also like to present some results about the relationship between realized betas and future returns. Lewellen (2011) is a recent example to analyze predictive FM regressions. We carry out the same exercise as in the previous section, but now the independent variables (the betas) and the dependent variable (the average monthly excess return of the stock) are calculated on different periods. At every month t, we estimate the conditional betas using data from the previous one-year period (months $t-11, ..., t$). These betas are then related to returns following month t. We consider three different horizons: next month's return ($t+1$), average return over the next three months ($t+1, ..., t+3$), and average return over the next six months ($t+1, ..., t+6$).

The results can be seen in Table 6. Our first observation is that the signs of all the lambda estimates remain the same as the ones we got when analyzing the contemporaneous relationship. Also, with one exception, the estimates remain significant at least at the 5% level. The only exception is $\lambda_{\mathcal{D}}$. It is not statistically significant at the one month and the three months horizon, and only significant at the 10% level when considering 6 months horizon.

All in all, we can conclude that the changes are not dramatic when we consider future returns instead of contemporaneous returns in the Fama-MacBeth regressions.

4.3.2 Portfolios as base assets

Although Ang et al. (2010) argue that it is more efficient to use individual stocks in cross-sectional asset pricing tests than portfolios, most of the literature uses portfolios as base assets. Therefore, we also analyze the empirical performance of our model using portfolios as base assets (with otherwise unchanged methodology). Data on the returns of our test portfolios is obtained from Kenneth R. French's data library[11].

We use the value-weighted return series of four different sets of portfolios: (i) 25 (5×5) portfolios formed on size and book-to-market, (ii) 25 (5×5) portfolios formed on size and momentum, (iii) 30 portfolios consisting of 10 size, 10 book-to-market, and 10 momentum portfolios, and (iv) 30

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

industry portfolios. Results of the FM regressions are presented in Table 7, while Figure 1 and Figure 2 show scatter plots of actual versus predicted returns for the different models and sets of portfolios.

The first observation is that the signs on the lambda estimates in our model (both for GDA3 and GDA5) remain the same as the ones when analyzing the contemporaneous relationship using individual stocks (the only exception is $\lambda_{\mathcal{D}}$ for the 25 Size-Momentum portfolios). Our second comment is about the statistical significance of the estimates: three factors, λ_W, $\lambda_{W\mathcal{D}}$ and $\lambda_{X\mathcal{D}}$ are statistically significant more or less consistently across all sets of portfolios. The other two factors ($\lambda_{\mathcal{D}}$ and λ_X) are generally not statistically significant. The magnitudes of the estimates are comparable to those in Table 5. The overall conclusion is that two factors loose their statistical significance if portfolios are considered in the Fama-MacBeth procedure instead of individual stocks. Other than that, the results seem to be quite robust to the choice of base assets.

The sum of squared pricing errors (labelled with "SSE") in Table 7 and Figures 1 - 2 describe the fit of the models. We can conclude that the GDA3 model is a considerable improvement compared to the standard CAPM, while the GDA5 model provides further considerable improvement. While the best fit (lowest SSE) is provided by the Carhart (1997) model for all the four sets of portfolios, the fit of the GDA5 model is comparable to that.

Decomposing returns

Using portfolios as base assets also allows us to decompose realized returns of these portfolios into parts that can be attributed to different factors. We carry out the following exercise: for each month t we have the $\hat{\beta}_{f_j,t}$ estimates from the first stage of the Fama-MacBeth procedure, and the $\hat{\lambda}_{f_j,t}$ estimates from the second stage of the FM procedure. The product $\hat{\beta}_{f_j,t} \cdot \hat{\lambda}_{f_j,t}$ is the part of the return at time t that can be attributed to factor f_j. We average these products across the whole sample period to arrive at the decomposition of the average returns of the portfolios.

Figure 3 shows the results of this exercise when the set of base assets consists of 10 portfolios sorted on size (S1 to S10), 10 portfolios sorted on book-to-market (B1 to B10), and finally 10 portfolios sorted on momentum (M1 to M10)[12]. Note, that during the estimation the 30 portfolios are considered at the same time, so the corresponding $E[\lambda_{f,t}]$ estimates are those presented in the lower-left panel of Table 7.

Let us first look at the results of the standard CAPM (top row in Figure 3). We can see that the predicted returns increase from the small portfolio (S1) to the portfolio of big firms (S10), while for the actual returns the relationship is reversed. If we look at the book-to-market portfolios, we can see that predicted returns are rather flat across the portfolios (B1 to B10), while realized returns show that value stocks (B10) outperform growth stocks (B1). In the case of the momentum portfolios, predicted returns increase from the looser portfolio (M1) to the winner portfolio (M10). This is the same pattern that can be observed in the actual data, However, the spread in the predicted returns is much smaller than the spread in the actual returns. These observations represent the failure of the standard CAPM in pricing these portfolios.

Fama and French (1993) developed the size (SMB) and value (HML) asset pricing factors to address this failure of the CAPM. Carhart (1997) added the momentum factor (WML) to the model. Results of this model are presented in the second row of Figure 3. The model provides a much improved fit compared to the standard CAPM. The predicted returns show the same patterns as the actual returns: they decrease from S1 to S10, and increase from B1 to B10 and M1 to M10. However, we would like to highlight one important observation: the improved fit of the size portfolios comes solely from the SMB factor. The improved fit of the B/M portfolios comes mostly from the HML factor. Finally, the improved fit of the momentum portfolios is almost exclusively due to the WML factor. These observations show how each factor was tailor made to explain its respective anomaly.

Let us look at the results of our GDA5 model in the bottom row of Figure 3. This model also provides a much better fit than the standard CAPM, and it is very similar to the Carhart (1997) model. The improvement for all three sets of portfolios is mainly coming from the contribution of two sources: the premium associated with the put option on the market return and the premium associated with the call option on the changes in market volatility. As it has been pointed out earlier,

[12]The same exercise can be carried out for the other sets of portfolios. The conclusions from analysing those sets of portfolios are similar to those presented here. The results are available upon request.

these premiums are required if and only if the investor is both risk averse and disappointment averse. The important observation that we would like highlight is that the same factors provide large part of the improvement across all portfolios.

4.3.3 Further robustness checks

Changing the definition of the disappointing event

So far in the empirical analysis, we have considered only one definition for the disappointing event: the market return falling below zero ($r_{W,t} < 0$). In this section we examine what happens if this definition is changed. Results are reported in Table 8. Recall, that the disappointing event corresponds to $r_{W,t} < \ln(\kappa/\delta)$ in the case of the GDA3 model, and to $r_{W,t} + (1/\psi)\,\varphi_{V\sigma}\Delta\sigma^2_{W,t} < \ln(\kappa/\delta^*)$ for the GDA5 model. In order to analyze different scenarios, we redefine the disappointing region the following way:

$$r_{W,t} - a\,\frac{\sigma_{r_W}}{\sigma_{\Delta\sigma^2_W}}\Delta\sigma^2_{W,t} < b\ .$$

Our baseline specification (the one that we have studied so far) corresponds to $a = b = 0$ (column 1 in Table 8). The disappointing region in the GDA3 model assumes $a = 0$. Column 2 in Table 8 presents the case when $a = 0$ and $b = -0.005$ [13]. In an average one-year period, the disappointing event occurs on 21% of the trading days with these parameter values, as opposed to 46% in our baseline case. Comparing the lambda estimates in the first two columns, we can see very little change. The rest of Table 8 presents scenarios when a is different from zero. We use the scaling factor $\frac{\sigma_{r_W}}{\sigma_{\Delta\sigma^2_W}}$, so that $r_{W,t}$ and $\Delta\sigma^2_{W,t}$ become comparable in magnitude [14]. The overall conclusion from Table 8 is that our baseline results are remarkably robust to changes in the definition of the disappointing event.

[13] This implies that $\frac{\kappa}{\delta} = 0.995$ from $r_{W,t} < \ln(\kappa/\delta)$.

[14] The value of the $\frac{\sigma_{r_W}}{\sigma_{\Delta\sigma^2_W}}$ ratio is around 800 in an average one-year period. We would also like to note that the typical value of $(1/\psi)\,\varphi_{V\sigma}$ in our calibrated model (see Table 1) is around -2600. This roughly corresponds to the case when $a = 3$.

Different measures of market volatility

In this section we explore how the results change if different measures of market volatility are considered. In our baseline specification (the results presented so far) daily market variance is estimated by fitting an EGARCH model to the daily market return series. Alternative approaches include using the VIX index, calculating daily realized variance from intra-daily market returns, or fitting a different model with conditional heteroskedasticity. Results using these approaches are presented in Table 9. For detailed description of the estimation of market variance in the different cases, we refer the reader to Appendix B.

Panel A of Table 9 presents the results using the whole sample period. Since the VIX and the intra-daily market return data is available after 1986, only the results for the model based approaches are presented. The results are very robust, there are only minor changes across the different GARCH specifications. The models that take into account the leverage effect (the EGARCH and the GJR-GARCH) perform slightly better than the standard GARCH. Panel B presents the results for the subsample between 1986 and 2010 when data is available for all the volatility measurement approaches. The adjusted R^2-s, the signs and statistical significance of the estimates are very similar across the specifications. The only difference is that the magnitudes of λ_X and $\lambda_{X\mathcal{D}}$ are higher when the VIX and the realized variance is used. The overall conclusion is that while we get the best results (in terms of adjusted R^2) when the EGARCH model is used, these results do not change much if a different approach is considered for measuring market volatility.

5 Revisiting measures of downside risk

In this section, we argue and show that exposures of asset payoffs to the three option factors provide a rational interpretation of downside risks. To achieve this, we show how our multivariate betas from equation (29) are related to a number of different measures put forward in previous empirical research to capture the market downside risk of an asset. We refer the reader to Appendix A.2 and A.3 for complete derivations of these relations.

One of the most popular measures of the market downside risk is the downside beta examined by Ang, Chen and Xing (2006) and defined as follows[15]:

$$\beta_i^{DM} \equiv \frac{Cov\left(R_{i,t}^e, r_{W,t} \mid \mathcal{D}_t\right)}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]}. \tag{37}$$

We show that

$$\beta_i^{DM} = \beta_{iW} + \beta_{iW\mathcal{D}} + \beta_X^{DM}\left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) \quad \text{where} \quad \beta_X^{DM} \equiv \frac{Cov\left(\Delta\sigma_{W,t}^2, r_{W,t} \mid \mathcal{D}_t\right)}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]}. \tag{38}$$

The above formula reduces to $\beta_i^{DM} = \beta_{iW} + \beta_{iW\mathcal{D}}$ for the GDA3 model. For the GDA5 model, we find empirically that the term $\beta_X^{DM}\left(\beta_{iX} + \beta_{iX\mathcal{D}}\right)$ is negligible so that $\beta_i^{DM} \approx \beta_{iW} + \beta_{iW\mathcal{D}}$. We report in Table 10 a sample cross-sectional correlation of 0.993 between β_i^{DM} and $\beta_{iW} + \beta_{iW\mathcal{D}}$ for the GDA5 model.

Similar to the market downside beta, we can introduce a volatility downside beta which we define by

$$\beta_i^{DV} \equiv \frac{Cov\left(R_{i,t}^e, \Delta\sigma_{W,t}^2 \mid \mathcal{D}_t\right)}{Var\left[\Delta\sigma_{W,t}^2 \mid \mathcal{D}_t\right]}. \tag{39}$$

We show that

$$\beta_i^{DV} = \beta_{iX} + \beta_{iX\mathcal{D}} + \beta_W^{DV}\left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) \quad \text{where} \quad \beta_W^{DV} \equiv \frac{Cov\left(r_{W,t}, \Delta\sigma_{W,t}^2 \mid \mathcal{D}_t\right)}{Var\left[\Delta\sigma_{W,t}^2 \mid \mathcal{D}_t\right]}. \tag{40}$$

We find empirically that the term $\beta_W^{DV}\left(\beta_{iW} + \beta_{iW\mathcal{D}}\right)$ is negligible so that $\beta_i^{DV} \approx \beta_{iX} + \beta_{iX\mathcal{D}}$. Our results from the previous section show that these two major components of the volatility downside beta are priced and predict future returns.

Post et al. (2010) advocate to use the semi-variance (SV) beta to measure the market downside risk. They study how realized market downside risk measures are related to future returns, and argue that the SV beta captures downside market risk better than the downside beta. The SV beta, that emerged from the lower partial moment framework of Bawa and Lindenberg (1977), is

[15]Empirically, Ang, Chen and Xing (2006) define the disappointing event as $r_{W,t} < \mu_W$, i.e. the market return falls below its long-run mean. Although we keep our definition of the disappointing event as closed as possible to the theory, the difference is empirically irrelevant.

defined by[16]

$$\beta_i^{SV} \equiv \frac{E\left[R_{i,t}^e r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]}. \tag{41}$$

We show that

$$\beta_i^{SV} = a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}} + a_X \beta_{iX} + a_{X\mathcal{D}} \beta_{iX\mathcal{D}} + a_R E\left[R_{i,t}^e\right] \tag{42}$$

where the a's coefficients are defined by

$$\begin{aligned} &a_W \equiv 1 - a_R E\left[r_{W,t}\right], \quad a_{W\mathcal{D}} \equiv 1 - \pi_1 a_R E\left[r_{W,t} \mid \mathcal{D}_t\right], \quad a_{\mathcal{D}} \equiv \left(1-\pi_1\right) a_R \\ &a_X \equiv \beta_X^{SV} - a_R E\left[\Delta\sigma_{W,t}^2\right], \quad a_{X\mathcal{D}} \equiv \beta_X^{SV} - \pi_1 a_R E\left[\Delta\sigma_{W,t}^2 \mid \mathcal{D}_t\right] \end{aligned} \tag{43}$$

and where

$$a_R \equiv \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]}, \quad \pi_1 = Prob\left(\mathcal{D}_t\right) \quad \text{and} \quad \beta_X^{SV} \equiv \frac{E\left[\Delta\sigma_{W,t}^2 r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]}. \tag{44}$$

Equation (42) reduces to $\beta_i^{SV} = a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}} + a_R E\left[R_{i,t}^e\right]$ for the GDA3 model.

Acharya et al. (2010) and Brownlees and Engle (2011) has used the Marginal Expected Shortfall (MES) to measure the systemic risk of financial institutions during the recent crisis. They show that the MES, together with the leverage of the institution, is able to predict emerging risks during the financial crisis.[17] We believe that it would be useful to show that the MES can be expressed in terms of exposures to the theoretical factors that are priced at the market place. The MES of an asset may be defined as

$$MES_i \equiv E\left[-R_{i,t}^e \mid \mathcal{D}_t\right]. \tag{45}$$

[16] Post et al. (2010) define the disappointing event similarly as $r_{W,t} < 0$, i.e. the market return is negative. As already mentioned, empirical results are robust to this alternative definition of the downside event.

[17] Since these authors focus on systemic risk, that is a worse downside risk, they empirically consider a disappointing event that is more infrequent compared to other papers on downside risks (5% worst days for the market return).

We show that

$$MES_i = a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}} + a_X \beta_{iX} + a_{X\mathcal{D}} \beta_{iX\mathcal{D}} + E\left[-R^e_{i,t}\right], \tag{46}$$

where the a's coefficients are defined by

$$\begin{aligned} &a_W \equiv -\left(E\left[r_{W,t} \mid \mathcal{D}_t\right] - E\left[r_{W,t}\right]\right), \quad a_{W\mathcal{D}} \equiv -\left(1-\pi_1\right) E\left[r_{W,t} \mid \mathcal{D}_t\right], \quad a_{\mathcal{D}} \equiv -\left(1-\pi_1\right) \\ &a_X \equiv -\left(E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] - E\left[\Delta\sigma^2_{W,t}\right]\right), \quad a_{X\mathcal{D}} \equiv -\left(1-\pi_1\right) E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]. \end{aligned} \tag{47}$$

Equation (46) reduces to $MES_i = a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}} + E\left[-R^e_{i,t}\right]$ for the GDA3 model.

Note, that the a's coefficients in equation (43) and (47) do not vary in the cross-section, so the variation of β_i^{SV} and MES_i across stocks is a result of the variation in the betas and the expected return of the asset. However, the relative magnitude of the weights may vary through time. Also observe that, since the SV beta and the MES not only contains our multivariate betas but also the mean excess return of the stock, they should not be applied in portfolio sorts when analyzing the contemporaneous relationship between downside risk measures and expected returns. Also, when the relationship between these two measures and future returns is analyzed (as in Post et al.; 2010 for the SV beta), the momentum effect is incorporated in the measure through the $E\left[R^e_{i,t}\right]$ term.

For empirical studies on downside risks and expected returns, we advocate using the relative SV beta and the relative MES which we define by

$$\beta_i^{RSV} \equiv \frac{E\left[R^e_{i,t} r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]} - \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]} E\left[R^e_{i,t}\right] \quad \text{and} \quad RMES_i \equiv E\left[-R^e_{i,t} \mid \mathcal{D}_t\right] - E\left[-R^e_{i,t}\right]. \tag{48}$$

The RMES of an asset is just equivalent to the opposite of its relative downside potential as previously defined in Section 4. The sample cross-sectional correlation between the original SV beta and the relative SV beta is 0.959, and the sample cross-sectional correlation between the marginal expected shortfall and the opposite of the relative downside potential is equal to 0.897 as reported in Table 10. The term $a_X \beta_{iX} + a_{X\mathcal{D}} \beta_{iX\mathcal{D}}$ is empirically irrelevant for the relative SV beta and the relative MES. Table 10 shows that the correlation between β_i^{RSV} and $a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}}$

is 0.995, while the correlation between the relative MSE and $a_W\beta_{iW} + a_{W\mathcal{D}}\beta_{iW\mathcal{D}} + a_{\mathcal{D}}\beta_{i\mathcal{D}}$ is 0.998 for the GDA5 model.

Finally, we observe from the a's coefficients expressed in equations (43) and (47) that a_W, $a_{W\mathcal{D}}$ and $a_{\mathcal{D}}$ are positive while a_X and $a_{X\mathcal{D}}$ are negative. This shows that, both the relative SV beta and the relative MES increase with the betas on the market return, the put option and the cash-or-nothing option, and decrease with the betas on changes in market volatility and the call option. An empirical investigation of how the a's coefficients vary through time and how they weight the different components of downside risks is left out for further research. While exposure to the cash-or-nothing option influences the relative SV beta and the relative MES, it plays no role in determining the market downside beta and the volatility downside beta.

6 Conclusion and Future Work

This paper provides an empirical analysis of downside risks in asset prices. The approach is consistent with general equilibrium implications for asset returns in the cross-section when investors have totally rational and axiomatized asymmetric preferences. The theoretical setup explicitly disentangles the components of an asset premium that are due to the different characteristics of investors' behavior, and shows that asymmetric preferences lead to option pricing in the cross-section of stock returns. These options provide a straightforward way for investors to act on their views of two of the most closely followed market variables, the market return and changes in market volatility. Empirical results show that the cross-section of stock returns reflects a premium for bearing undesirable exposures to these options, and that the new cross-sectional model significantly improves over nested specifications without the option factors.

The paper also derives explicit cross-sectional relations between existing downside risk measures and betas on the market return, changes in market volatility and option factors. The weights associated to these relations and how they vary through time and in relation with the business cycle may constitute an interesting avenue for future research.

Apppendix

A Relationships between different measures

A.1 Relating $\sigma_{i\mathcal{D}}$ to $E\left[R^e_{i,t} \mid \mathcal{D}_t\right]$

$\sigma_{i\mathcal{D}}$ can be expressed as

$$\begin{aligned}\sigma_{i\mathcal{D}} &= Cov\left[R^e_{i,t}, I\left(\mathcal{D}_t\right)\right] = E\left[R^e_{i,t} I\left(\mathcal{D}_t\right)\right] - E\left[R^e_{i,t}\right] E\left[I\left(\mathcal{D}_t\right)\right] \\ &= E\left[R^e_{i,t}|\mathcal{D}_t\right] P\left(\mathcal{D}_t\right) - E\left[R^e_{i,t}\right] P\left(\mathcal{D}_t\right) \\ &= P\left(\mathcal{D}_t\right)\left(E\left[R^e_{i,t}|\mathcal{D}_t\right] - E\left[R^e_{i,t}\right]\right) \ .\end{aligned} \tag{A.1}$$

Note, that $P\left(\mathcal{D}_t\right)$ does not vary in the cross-section, so sorting on $\sigma_{i\mathcal{D}}$ is equivalent to sorting on $E\left[R^e_{i,t}|\mathcal{D}_t\right] - E\left[R^e_{i,t}\right]$.

A.2 The GDA3 model

Let us start with the GDA3 model. The beta measures are calculated using daily data from $t = 1, ..., T$ with the following OLS regression.

$$R^e_{i,t} = \alpha_i + \beta_{iW} \cdot r_{W,t} + \beta_{iW\mathcal{D}} \cdot r_{W,t} I(\mathcal{D}_t) + \beta_{i\mathcal{D}} \cdot I(\mathcal{D}_t) + \varepsilon_{it} \tag{A.2}$$

The mechanics of the OLS give us the following four equations

$$\begin{aligned}0 &= E\left[\varepsilon_{it}\right] \\ E\left[R^e_{i,t}\right] &= \alpha_i + \beta_{i\mathcal{D}} P\left(\mathcal{D}_t\right) + \beta_{iW} E\left[r_{W,t}\right] + \beta_{iW\mathcal{D}} E\left[r_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right)\end{aligned} \tag{A.3}$$

$$\begin{aligned}0 &= E\left[r_{W,t} \cdot \varepsilon_{it}\right] \\ 0 &= E\left[r_{W,t}\left(R^e_{i,t} - \alpha_i - \beta_{iW} \cdot r_{W,t} - \beta_{iW\mathcal{D}} \cdot r_{W,t} I(\mathcal{D}_t) - \beta_{i\mathcal{D}} \cdot I(\mathcal{D}_t)\right)\right] \\ E\left[R^e_{i,t} r_{W,t}\right] &= \alpha_i E\left[r_{W,t}\right] + \beta_{i\mathcal{D}} E\left[r_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) + \beta_{iW} E\left[r^2_{W,t}\right] + \beta_{iW\mathcal{D}} E\left[r^2_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right)\end{aligned} \tag{A.4}$$

$$\begin{aligned}0 &= E\left[r_{W,t} I(\mathcal{D}_t) \cdot \varepsilon_{it}\right] \\ 0 &= E\left[r_{W,t} I(\mathcal{D}_t)\left(R^e_{i,t} - \alpha_i - \beta_{iW} \cdot r_{W,t} - \beta_{iW\mathcal{D}} \cdot r_{W,t} I(\mathcal{D}_t) - \beta_{i\mathcal{D}} \cdot I(\mathcal{D}_t)\right)\right] \\ E\left[R^e_{i,t} r_{W,t} I(\mathcal{D}_t)\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) E\left[r_{W,t} I(\mathcal{D}_t)\right] + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r^2_{W,t} I(\mathcal{D}_t)\right] \\ E\left[R^e_{i,t} r_{W,t} \mid \mathcal{D}_t\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) E\left[r_{W,t} \mid \mathcal{D}_t\right] + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r^2_{W,t} \mid \mathcal{D}_t\right]\end{aligned} \tag{A.5}$$

$$\begin{aligned}0 &= E\left[I(\mathcal{D}_t) \cdot \varepsilon_{it}\right] \\ 0 &= E\left[I(\mathcal{D}_t)\left(R^e_{i,t} - \alpha_i - \beta_{iW} \cdot r_{W,t} - \beta_{iW\mathcal{D}} \cdot r_{W,t} I(\mathcal{D}_t) - \beta_{i\mathcal{D}} \cdot I(\mathcal{D}_t)\right)\right] \\ E\left[R^e_{i,t} I(\mathcal{D}_t)\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) E\left[I(\mathcal{D}_t)\right] + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r_{W,t} I(\mathcal{D}_t)\right] \\ E\left[R^e_{i,t} \mid \mathcal{D}_t\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r_{W,t} \mid \mathcal{D}_t\right]\end{aligned} \tag{A.6}$$

The **downside beta** used in Ang, Chen and Xing (2006) is defined as

$$\beta_i^{DM} = \frac{Cov\left[R_{i,t}^e, r_{W,t} \mid \mathcal{D}_t\right]}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} \tag{A.7}$$

Using (A.5) and (A.6), this can be rewritten as

$$\begin{aligned}
\beta_i^{DM} &= \frac{E\left[R_{i,t}^e r_{W,t} \mid \mathcal{D}_t\right] - E\left[R_{i,t}^e \mid \mathcal{D}_t\right] E\left[r_{W,t} \mid \mathcal{D}_t\right]}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} \\
&= \frac{\left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) \cdot \left(E\left[r_{W,t}^2 \mid \mathcal{D}_t\right] - E^2\left[r_{W,t} \mid \mathcal{D}_t\right]\right)}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} \\
&= \beta_{iW} + \beta_{iW\mathcal{D}}
\end{aligned} \tag{A.8}$$

The **semi-variance beta** used by Post et al. (2010) is defined as

$$\beta_i^{SV} = \frac{E\left[R_{i,t}^e r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]} \tag{A.9}$$

Using (A.5), this can be rewritten as

$$\beta_i^{SV} = \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]} \cdot \left(\alpha_i + \beta_{i\mathcal{D}}\right) + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) \tag{A.10}$$

Substituting for α_i using (A.3) will result in

$$\begin{aligned}
\beta_i^{SV} = E\left[R_{i,t}^e\right] \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]} + \beta_{i\mathcal{D}} \left(1 - P\left(\mathcal{D}_t\right)\right) \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]} + \\
+ \beta_{iW} \left(1 - E\left[r_{W,t}\right] \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]}\right) + \beta_{iW\mathcal{D}} \left(1 - P\left(\mathcal{D}_t\right) \frac{E^2\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r_{W,t}^2 \mid \mathcal{D}_t\right]}\right)
\end{aligned} \tag{A.11}$$

The **marginal expected shortfall** from Brownlees and Engle (2011) is simply the negative of (A.6),

$$MES_i = -E\left[R_{i,t}^e \mid \mathcal{D}_t\right] = -\left(\alpha_i + \beta_{i\mathcal{D}}\right) - E\left[r_{W,t} \mid \mathcal{D}_t\right] \cdot \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) \tag{A.12}$$

Substituting for α_i using (A.3), will result in

$$MES_i = \left(P\left(\mathcal{D}_t\right) - 1\right) \beta_{i\mathcal{D}} + \left(E\left[r_{W,t}\right] - E\left[r_{W,t} \mid \mathcal{D}_t\right]\right) \beta_{iW} + E\left[r_{W,t} \mid \mathcal{D}_t\right] \left(P\left(\mathcal{D}_t\right) - 1\right) \beta_{iW\mathcal{D}} - E\left[R_{i,t}^e\right] \tag{A.13}$$

A.3 The GDA5 model

Let us look at the GDA5 model. The betas are calculated from the following OLS regression

$$R_{i,t}^e = \alpha_i + \beta_{iW} \cdot r_{W,t} + \beta_{iW\mathcal{D}} \cdot r_{W,t} I(\mathcal{D}_t) + \beta_{i\mathcal{D}} \cdot I(\mathcal{D}_t) + \beta_{iX} \cdot \Delta\sigma_{W,t}^2 + \beta_{iX\mathcal{D}} \cdot \Delta\sigma_{W,t}^2 I(\mathcal{D}_t) + \varepsilon_t^i \tag{A.14}$$

Similarly to the previous section, the mechanics of the OLS give us the following results:

$$\begin{aligned} E\left[R^e_{i,t}\right] &= \alpha_i + \beta_{i\mathcal{D}} P\left(\mathcal{D}_t\right) + \beta_{iW} E\left[r_{W,t}\right] + \beta_{iW\mathcal{D}} E\left[r_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) + \\ &+ \beta_{iX} E\left[\Delta\sigma^2_{W,t}\right] + \beta_{iX\mathcal{D}} E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) \end{aligned} \tag{A.15}$$

$$\begin{aligned} E\left[R^e_{i,t} r_{W,t}\right] &= \alpha_i E\left[r_{W,t}\right] + \beta_{i\mathcal{D}} E\left[r_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) + \beta_{iW} E\left[r^2_{W,t}\right] + \beta_{iW\mathcal{D}} E\left[r^2_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) + \\ &+ \beta_{iX} E\left[r_{W,t}\Delta\sigma^2_{W,t}\right] + \beta_{iX\mathcal{D}} E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) \end{aligned} \tag{A.16}$$

$$\begin{aligned} E\left[R^e_{i,t} r_{W,t} \mid \mathcal{D}_t\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) E\left[r_{W,t} \mid \mathcal{D}_t\right] + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r^2_{W,t} \mid \mathcal{D}_t\right] + \\ & \left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] \end{aligned} \tag{A.17}$$

$$E\left[R^e_{i,t} \mid \mathcal{D}_t\right] = \left(\alpha_i + \beta_{i\mathcal{D}}\right) + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r_{W,t} \mid \mathcal{D}_t\right] + \left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] \tag{A.18}$$

$$\begin{aligned} E\left[R^e_{i,t}\Delta\sigma^2_{W,t}\right] &= \alpha_i E\left[\Delta\sigma^2_{W,t}\right] + \beta_{i\mathcal{D}} E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) + \beta_{iW} E\left[r_{W,t}\Delta\sigma^2_{W,t}\right] + \\ &+ \beta_{iW\mathcal{D}} E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) + \beta_{iX} E\left[\left(\Delta\sigma^2_{W,t}\right)^2\right] + \\ &+ \beta_{iX\mathcal{D}} E\left[\left(\Delta\sigma^2_{W,t}\right)^2 \mid \mathcal{D}_t\right] P\left(\mathcal{D}_t\right) \end{aligned} \tag{A.19}$$

$$\begin{aligned} E\left[R^e_{i,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] + \\ &+ \left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) E\left[\left(\Delta\sigma^2_{W,t}\right)^2 \mid \mathcal{D}_t\right] \end{aligned} \tag{A.20}$$

The **downside beta** used in Ang, Chen and Xing (2006) is defined as in (A.7). Using (A.17) and (A.18), it can be rewritten as

$$\begin{aligned} \beta_i^{DM} &= \frac{E\left[R^e_{i,t} r_{W,t} \mid \mathcal{D}_t\right] - E\left[R^e_{i,t} \mid \mathcal{D}_t\right] E\left[r_{W,t} \mid \mathcal{D}_t\right]}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} \\ &= \frac{\left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) \cdot \left(E\left[r^2_{W,t} \mid \mathcal{D}_t\right] - E^2\left[r_{W,t} \mid \mathcal{D}_t\right]\right)}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} + \\ &\quad + \frac{\left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) \cdot \left(E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right)\right] - E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] E\left[r_{W,t} \mid \mathcal{D}_t\right]\right)}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} \\ &= \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) + \frac{Cov\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]} \cdot \left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) \end{aligned} \tag{A.21}$$

The **semi-variance beta** used by Post et al. (2010) is defined as in (A.9). Using (A.17), this can be rewritten as

$$\beta_i^{SV} = \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]} \cdot \left(\alpha_i + \beta_{i\mathcal{D}}\right) + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) + \frac{E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]} \cdot \left(\beta_{iX} + \beta_{iX\mathcal{D}}\right) \tag{A.22}$$

The **marginal expected shortfall** from Brownlees and Engle (2011) is simply the negative of (A.18),

$$MES_i = -(\alpha_i + \beta_{i\mathcal{D}}) - E[r_{W,t} \mid \mathcal{D}_t] \cdot (\beta_{iW} + \beta_{iW\mathcal{D}}) - E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] \cdot (\beta_{iX} + \beta_{iX\mathcal{D}}) \tag{A.23}$$

Substituting for α_i using (A.15), will result in

$$\begin{aligned} MES_i = {} & (P(\mathcal{D}_t) - 1)\,\beta_{i\mathcal{D}} + (E[r_{W,t}] - E[r_{W,t} \mid \mathcal{D}_t])\,\beta_{iW} + E[r_{W,t} \mid \mathcal{D}_t]\,(P(\mathcal{D}_t) - 1)\,\beta_{iW\mathcal{D}} \\ & + \left(E\left[\Delta\sigma^2_{W,t}\right] - E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]\right)\beta_{iX} + E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right](P(\mathcal{D}_t) - 1)\,\beta_{iX\mathcal{D}} - E\left[R^e_{i,t}\right] \end{aligned} \tag{A.24}$$

The **volatility downside beta** can be defined similarly to the market downside beta:

$$\beta_i^{DV} \equiv \frac{Cov\left[R^e_{i,t}, \Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}{Var\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]} \tag{A.25}$$

Using (A.18) and (A.20), this can be rewritten as

$$\begin{aligned} \beta_i^{DV} &= \frac{E\left[R^e_{i,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right] - E\left[R^e_{i,t} \mid \mathcal{D}_t\right]E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}{Var\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]} \\ &= \frac{(\beta_{iX} + \beta_{iX\mathcal{D}}) \cdot \left(E\left[\left(\Delta\sigma^2_{W,t}\right)^2 \mid \mathcal{D}_t\right] - E^2\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]\right)}{Var\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]} + \\ &\quad + \frac{(\beta_{iW} + \beta_{iW\mathcal{D}}) \cdot \left(E\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t)\right] - E\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]E[r_{W,t} \mid \mathcal{D}_t]\right)}{Var\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]} \\ &= (\beta_{iX} + \beta_{iX\mathcal{D}}) + \frac{Cov\left[r_{W,t}\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}{Var\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]} \cdot (\beta_{iW} + \beta_{iW\mathcal{D}}) \end{aligned} \tag{A.26}$$

B Different measures of market volatility

B.1 VIX

The daily value of the VIX index is obtained from CBOE through the WRDS service. The variance of the market is calculated as $\left(\frac{VIX}{100}\right)^2$. Since the VIX measures 30-day expected volatility of the S&P 500 Index, we divide this value by 30 to get the daily variance of the market. So, the change in the daily market variance is calculated as

$$\Delta\sigma_{W,t}^{2,VIX} = \frac{\left(VIX_t/100\right)^2 - \left(VIX_{t-1}/100\right)^2}{30} \tag{B.1}$$

B.2 Realized Volatility

To calculate daily realized volatility, we use intra-daily return series of the S&P 500. The data comes from Olsen Financial Technologies and covers the period between February 1986 and September 2010. Daily realized market variance is calculated as

$$\sigma_{W,t}^{2,RV} = \sum_j r_{j,t}^2 \ , \tag{B.2}$$

where $r_{j,t}$ denotes the 10-minute log return series on the trading day t. Following Bandi et al. (2006) we correct the variance estimates for the lack of overnight returns by multiplying them with a constant factor $\xi = \frac{\frac{1}{T}\sum_{t=1}^{T} r_{W,t}^2}{\frac{1}{T}\sum_{t=1}^{T} \sigma_{W,t}^{2,RV}}$, where $r_{W,t}$ denotes daily log returns on the market. The change in the daily market variance is calculated as

$$\Delta\sigma_{W,t}^{2,RV} = \xi\left(\sigma_{W,t}^{2,RV} - \sigma_{W,t-1}^{2,RV}\right) \tag{B.3}$$

B.3 GARCH type of models

In this approach, we fit a model with conditional heteroskedasticity to the daily log market return series $r_{W,t}$ (the value-weighted average return on all NYSE, AMEX, and NASDAQ stocks from CRSP). We consider three different models: the standard GARCH(1,1), the EGARCH(1,1,1) by Nelson (1991) and the GJR-GARCH(1,1,1) by Glosten et al. (1993). The models are given as (the difference is in the variance equation):

$$\begin{aligned}
r_{W,t} &= \mu + \epsilon_t \ , \quad \text{with} \quad \epsilon_t = \sigma_{W,t} e_t \ , \quad \text{and} \quad e_t \overset{\text{iid}}{\sim} N(0,1) \\
GARCH: \ \sigma_{W,t}^2 &= \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{W,t-1}^2 \\
EGARCH: \ \ln\left(\sigma_{W,t}^2\right) &= \omega + \alpha\left(\left|\frac{\epsilon_{t-1}}{\sigma_{W,t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) + \gamma\frac{\epsilon_{t-1}}{\sigma_{W,t-1}} + \beta\ln\left(\sigma_{W,t-1}^2\right) \\
GJR-GARCH: \ \sigma_{W,t}^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I\left(\epsilon_{t-1} < 0\right) + \beta\sigma_{W,t-1}^2
\end{aligned} \tag{B.4}$$

The change in the daily market variance is calculated as

$$\Delta\sigma_{W,t}^{2,model} = \hat{\sigma}_{W,t}^{2,model} - \hat{\sigma}_{W,t-1}^{2,model} \tag{B.5}$$

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Table 1: **Model-Implied Values: Five-Factor Cross-Section Risk-Return Tradeoff**

	Model Calibration Scenarios								
δ	0.9979	0.9979	0.9979	0.9979	0.9969	0.9969	0.9969	0.9969	0.9969
γ	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75
ψ	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
α	0.3	0.3	0.2	0.4	0.3	0.3	0.3	0.1	0.1
κ	0.992	0.997	0.993	0.995	0.999	0.998	0.997	0.991	0.99
ϕ_σ	0.995	0.99	0.99	0.995	0.99	0.99	0.99	0.99	0.99
A.	Risk-Free Rate								
$E\left[r_f\right]$	1.31	0.75	0.74	1.41	1.81	1.87	1.97	1.24	1.32
$\sigma\left[r_f\right]$	2.46	1.98	3.22	1.69	1.74	1.85	1.98	4.70	4.78
B.	Welfare Valuation Ratio								
φ_{V0}	1.945	1.238	1.078	1.646	1.232	1.126	1.023	1.432	0.600
$\varphi_{V\sigma}$	-6.037E+3	-4.152E+3	-3.339E+3	-4.947E+3	-4.051E+3	-3.385E+3	-2.740E+3	-4.630E+3	-1.307E+3
$\varphi_{\mathcal{R}0}$	1.946	1.237	1.077	1.646	1.231	1.125	1.022	1.431	0.598
$\varphi_{\mathcal{R}\sigma}$	-6.043E+3	-4.153E+3	-3.338E+3	-4.953E+3	-4.054E+3	-3.388E+3	-2.743E+3	-4.628E+3	-1.307E+3
φ	1.00110	1.00016	0.99988	1.00130	1.00076	1.00100	1.00123	0.99950	0.99981
δ^*	0.99737	0.99842	0.99858	0.99745	0.99736	0.99729	0.99727	0.99820	0.99785
C.	Disappointment Probability								
$\pi_1\,(L)$	0.0122	0.0682	0.0088	0.0597	0.1246	0.0975	0.0745	0.0025	0.0024
$\pi_1\,(H)$	0.2105	0.2888	0.1951	0.2822	0.3172	0.3026	0.2881	0.1252	0.1183
$E\left[\pi_1\right]$	0.0541	0.1148	0.0482	0.1067	0.1653	0.1408	0.1197	0.0285	0.0269
D.	Factor Risk Premia								
$\lambda_W\,(L)$	0.0073	0.0072	0.0044	0.0057	0.0072	0.0055	0.0042	0.0052	0.0027
$\lambda_W\,(H)$	0.0077	0.0083	0.0104	0.0063	0.0083	0.0083	0.0083	0.0140	0.0139
$E\left[\lambda_W\right]$	0.0074	0.0075	0.0057	0.0058	0.0075	0.0061	0.0051	0.0071	0.0050
$\lambda_X\,(L)$	-1.19E-5	-1.37E-5	-9.85E-6	-8.07E-6	-1.25E-5	-9.13E-6	-6.67E-6	-1.42E-5	-7.59E-6
$\lambda_X\,(H)$	-1.09E-6	-2.08E-6	-1.87E-6	-1.10E-6	-2.09E-6	-2.01E-6	-1.90E-6	-1.79E-6	-1.44E-6
$E\left[\lambda_X\right]$	-9.58E-6	-1.12E-5	-8.16E-6	-6.60E-6	-1.03E-5	-7.63E-6	-5.66E-6	-1.16E-5	-6.29E-6
$\lambda_{\mathcal{D}}\,(L)$	-0.0744	-0.1779	-0.0666	-0.1160	-0.2421	-0.2022	-0.1637	-0.0676	-0.0373
$\lambda_{\mathcal{D}}\,(H)$	-0.2781	-0.3045	-0.3741	-0.2301	-0.3079	-0.3070	-0.3055	-0.4866	-0.4794
$E\left[\lambda_{\mathcal{D}}\right]$	-0.1175	-0.2046	-0.1316	-0.1401	-0.2560	-0.2243	-0.1937	-0.1561	-0.1307
$\lambda_{W\mathcal{D}}\,(L)$	0.0067	0.0064	0.0040	0.0051	0.0060	0.0046	0.0035	0.0047	0.0025
$\lambda_{W\mathcal{D}}\,(H)$	0.0036	0.0030	0.0041	0.0027	0.0028	0.0029	0.0031	0.0052	0.0053
$E\left[\lambda_{W\mathcal{D}}\right]$	0.0061	0.0057	0.0041	0.0046	0.0053	0.0042	0.0034	0.0048	0.0031
$\lambda_{X\mathcal{D}}\,(L)$	-1.19E-5	-1.37E-5	-9.85E-6	-8.06E-6	-1.25E-5	-9.12E-6	-6.66E-6	-1.42E-5	-7.59E-6
$\lambda_{X\mathcal{D}}\,(H)$	8.48E-8	1.46E-8	-1.24E-8	8.50E-8	7.04E-8	9.10E-8	1.09E-7	-6.21E-8	-2.30E-8
$E\left[\lambda_{X\mathcal{D}}\right]$	-9.33E-6	-1.08E-5	-7.77E-6	-6.34E-6	-9.83E-6	-7.17E-6	-5.23E-6	-1.12E-5	-5.99E-6

The table shows the model-implied expected annualized risk-free rate and its volatility in Panel A, the drift and loading coefficients of the welfare valuation ratio onto market volatility in Panel B, the disappointment probability and its unconditional value in Panel C, and finally the factor risk premia and their expected values in Panel D.

Table 2: Estimating the EGARCH model

The model we estimate - EGARCH(1,1,1)

$$
\begin{aligned}
r_{W,t} =& \mu + \epsilon_t \\
\epsilon_t =& \sigma_{W,t} e_t \\
\ln\left(\sigma^2_{W,t}\right) =& \omega + \alpha\left(\left|\frac{\epsilon_{t-1}}{\sigma_{W,t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) + \gamma\frac{\epsilon_{t-1}}{\sigma_{W,t-1}} + \beta\ln\left(\sigma^2_{W,t-1}\right) \\
e_t \overset{iid}{\sim}& N(0,1)
\end{aligned}
$$

Estimates

	μ	ω	α	γ	β
Coeff	4.13E-4	-0.141	0.150	-0.074	0.985
s.e.	(0.0001)	(0.0098)	(0.0050)	(0.0031)	(0.0010)

Table 3: Correlations between measures of risk

	σ_{iW}	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	σ_{iX}	$\sigma_{iX\mathcal{D}}$	β_{iW}	$\beta_{i\mathcal{D}}$	$\beta_{iW\mathcal{D}}$	β_{iX}	$\beta_{iX\mathcal{D}}$
σ_{iW}	1.00									
$\sigma_{i\mathcal{D}}$	-0.91	1.00								
$\sigma_{iW\mathcal{D}}$	0.94	-0.86	1.00							
σ_{iX}	-0.01	0.02	-0.01	1.00						
$\sigma_{iX\mathcal{D}}$	-0.03	0.01	-0.04	0.68	1.00					
β_{iW}	0.73	-0.51	0.52	-0.01	-0.01	1.00				
$\beta_{i\mathcal{D}}$	0.04	0.36	0.03	0.00	-0.02	0.42	1.00			
$\beta_{iW\mathcal{D}}$	-0.05	0.04	0.26	0.04	0.00	-0.58	-0.02	1.00		
β_{iX}	0.10	-0.08	0.10	0.72	0.06	0.07	0.01	0.01	1.00	
$\beta_{iX\mathcal{D}}$	-0.00	-0.01	-0.00	-0.01	0.68	-0.02	0.01	0.05	-0.65	1.00

The above table shows the correlation matrix of several measures of risk connected to our analysis. At every month t, we calculate the cross-sectional correlation of the measures estimated using daily data from the previous one-year period. The values presented in these tables are the time-series averages of these cross-sectional correlations over the sample period. The sample period is July, 1963 - December, 2010.

Table 4: Average returns of portfolios sorted on different measures of risk

Panel A: equal-weighted portfolios

	measures calculated separately					measures from GDA5 model				
	σ_{iW}	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	σ_{iX}	$\sigma_{iX\mathcal{D}}$	β_{iW}	$\beta_{i\mathcal{D}}$	$\beta_{iW\mathcal{D}}$	β_{iX}	$\beta_{iX\mathcal{D}}$
Low	5.10	17.44	4.28	15.29	15.79	9.28	11.39	5.70	12.59	12.96
2	7.60	11.21	7.33	11.75	12.34	8.73	9.80	7.84	10.89	11.15
3	9.47	9.65	9.00	9.64	10.02	9.57	9.08	9.61	9.38	9.67
4	11.33	7.62	11.23	7.84	7.79	10.73	9.60	11.77	8.62	9.12
High	17.99	5.51	19.81	6.76	5.42	12.81	11.25	16.44	9.65	8.25
H-L	**12.89**	**-11.93**	**15.54**	**-8.53**	**-10.37**	**3.53**	**-0.14**	**10.73**	**-2.94**	**-4.70**
t-stat	3.70	-3.61	4.16	-4.90	-7.35	1.70	-0.16	6.20	-2.02	-6.25

Panel B: value-weighted portfolios

	measures calculated separately					measures from GDA5 model				
	σ_{iW}	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	σ_{iX}	$\sigma_{iX\mathcal{D}}$	β_{iW}	$\beta_{i\mathcal{D}}$	$\beta_{iW\mathcal{D}}$	β_{iX}	$\beta_{iX\mathcal{D}}$
Low	6.22	12.36	6.12	15.48	16.76	8.02	10.57	5.18	12.10	13.22
2	6.60	8.10	6.31	12.29	13.22	7.81	9.07	8.37	9.50	10.98
3	7.34	7.38	7.12	9.48	10.30	8.13	8.33	9.54	8.30	8.50
4	8.06	6.47	8.82	7.35	7.53	8.38	8.25	11.83	8.25	7.34
High	12.73	6.23	14.78	5.96	4.48	9.90	8.92	15.93	8.95	5.68
H-L	**6.50**	**-6.13**	**8.66**	**-9.52**	**-12.28**	**1.88**	**-1.65**	**10.75**	**-3.15**	**-7.54**
t-stat	1.88	-1.78	2.30	-5.16	-6.34	0.72	-1.07	3.79	-2.10	-5.71

The table lists the equal-weighted (Panel A) and value-weighted (Panel B) average returns of stocks sorted by realized covariances and betas. For each month, σ-s and β-s are calculated using daily simple excess returns over the previous 12 months (including the given month). For each month and each risk measure, we rank stocks into 5 portfolios, and the average monthly excess returns (over the previous 12 months) of these portfolios are calculated. The table reports the annualized average return of these portfolios over the whole sample period. The row labelled "H-L" reports the difference between the returns of portfolio 5 and portfolio 1. The row labelled "t-stat" is the t-statistics computed using Newey-West (1987) standard errors with 12 lags for the H-L difference.

Table 5: Fama-Macbeth regressions on contemporaneous returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Cons	0.0037*		0.0027		0.0018		0.0028*		0.0020	
	(0.0019)		(0.0019)		(0.0018)		(0.0016)		(0.0016)	
λ_W	0.0061***	0.0087***	0.0062***	0.0081***	0.0065***	0.0077***	0.0048***	0.0064***	0.0055***	0.0067***
	(0.0017)	(0.0023)	(0.0016)	(0.0021)	(0.0017)	(0.0020)	(0.0012)	(0.0015)	(0.0012)	(0.0015)
$\lambda_{\mathcal{D}}$			-0.3193***	-0.4607***	-0.3317***	-0.4267***			-0.2888***	-0.3737***
			(0.0723)	(0.1008)	(0.0746)	(0.0957)			(0.0567)	(0.0714)
$\lambda_{W\mathcal{D}}$			0.0063***	0.0069***	0.0064***	0.0066***			0.0051***	0.0054***
			(0.0013)	(0.0015)	(0.0012)	(0.0014)			(0.0009)	(0.0011)
λ_X					-8.3E-6***	-5.8E-6			-7.8E-6***	-6.3E-6*
					(3.0E-6)	(4.1E-6)			(2.7E-6)	(3.5E-6)
$\lambda_{X\mathcal{D}}$					-6.9E-6***	-6.9E-6***			-6.2E-6***	-6.2E-6***
					(1.4E-6)	(1.9E-6)			(1.1E-6)	(1.4E-6)
λ_{SMB}							0.0029***	0.0033***	0.0030***	0.0032***
							(0.0011)	(0.0012)	(0.0011)	(0.0012)
λ_{HML}							-0.0007	-0.0007	-0.0008	-0.0008
							(0.0010)	(0.0010)	(0.0010)	(0.0010)
λ_{WML}							0.0111***	0.0113***	0.0114***	0.0116***
							(0.0013)	(0.0013)	(0.0013)	(0.0013)
adj R^2	0.0384		0.0524		0.0625		0.0985		0.1092	
Sharpe Ratios										
λ_W	0.6267		0.6402		0.6795		0.4910		0.5739	
$\lambda_{\mathcal{D}}$			0.4878		0.5069				0.4413	
$\lambda_{W\mathcal{D}}$			1.0593		1.0948				0.8886	
λ_X					0.7573				0.6773	
$\lambda_{X\mathcal{D}}$					0.8637				0.7394	
λ_{SMB}							0.5083		0.5214	
λ_{HML}							0.1135		0.1361	
λ_{WML}							1.6031		1.6425	

The Table presents results of Fama-MacBeth (1973) regressions. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months: $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. Where applicable, adjusted R^2 of the model is also reported.
In the lower panel average annualized Sharpe ratios of well-diversified single exposures to the given factors are presented.

Table 6: Fama-Macbeth regressions using future returns

	R_{t+1}			$R_{t+1,t+3}$			$R_{t+1,t+6}$		
λ_W	0.0054**	0.0050**	0.0047*	0.0053**	0.0050**	0.0046**	0.0053**	0.0050**	0.0046**
	(0.0026)	(0.0025)	(0.0024)	(0.0025)	(0.0024)	(0.0023)	(0.0024)	(0.0023)	(0.0022)
$\lambda_{\mathcal{D}}$		-0.1689	-0.1500		-0.1968*	-0.1767		-0.2073*	-0.1855*
		(0.1187)	(0.1107)		(0.1154)	(0.1082)		(0.1092)	(0.1031)
$\lambda_{W\mathcal{D}}$		0.0042**	0.0039**		0.0040**	0.0037**		0.0042**	0.0039**
		(0.0018)	(0.0017)		(0.0018)	(0.0017)		(0.0018)	(0.0017)
λ_X			-1.8E-5**			-1.8E-5***			-1.8E-5***
			(7.0E-6)			(6.9E-6)			(6.4E-6)
$\lambda_{X\mathcal{D}}$			-6.8E-6***			-7.5E-6***			-7.9E-6***
			(2.2E-6)			(2.3E-6)			(2.2E-6)

The Table presents results of Fama-MacBeth regressions. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the **next month** ($t+1$, labelled "R_{t+1}"), **next 3 months** ($t+1$ to $t+3$, labelled "$R_{t+1,t+3}$"), and **next 6 months** ($t+1$ to $t+6$, labelled "$R_{t+1,t+6}$"). The standard errors (in parenthesis) are corrected for 12 Newey-West lags.

Table 7: Fama-Macbeth regressions on portfolios

	25 Size - BM				25 Size - Momentum			
λ_W	0.0064***	0.0043***	0.0052***	0.0047***	0.0068***	0.0051***	0.0052***	0.0047***
	(0.0019)	(0.0016)	(0.0017)	(0.0016)	(0.0019)	(0.0017)	(0.0017)	(0.0017)
$\lambda_{\mathcal{D}}$			-0.3969	-0.2477			0.4210	0.2153
			(0.3625)	(0.3156)			(0.2892)	(0.2744)
$\lambda_{W\mathcal{D}}$			0.0112***	0.0067**			0.0158***	0.0103***
			(0.0032)	(0.0029)			(0.0033)	(0.0029)
λ_X				-1.6E-5				-3.3E-5*
				(1.1E-5)				(1.8E-5)
$\lambda_{X\mathcal{D}}$				-1.4E-5*				-3.6E-5***
				(8.2E-6)				(1.2E-5)
λ_{SMB}		0.0021				0.0025*		
		(0.0013)				(0.0013)		
λ_{HML}		0.0037***				0.0031		
		(0.0012)				(0.0019)		
λ_{WML}		0.0129***				0.0064***		
		(0.0022)				(0.0017)		
SSE	0.0017	0.0005	0.0009	0.0007	0.0023	0.0007	0.0013	0.0009

	10 Size, 10 BM, 10 Momentum				30 industry			
λ_{r_W}	0.0052***	0.0046***	0.0047***	0.0046***	0.0054***	0.0053***	0.0053***	0.0050***
	(0.0017)	(0.0016)	(0.0017)	(0.0016)	(0.0017)	(0.0016)	(0.0017)	(0.0016)
$\lambda_{I(\mathcal{D})}$			-0.2632	-0.2542			-0.4592**	-0.3828
			(0.2451)	(0.2128)			(0.2292)	(0.2361)
$\lambda_{r_W I(\mathcal{D})}$			0.0096***	0.0061**			0.0081***	0.0061***
			(0.0032)	(0.0027)			(0.0019)	(0.0019)
$\lambda_{\Delta\sigma^2_W}$				-1.3E-5				-9.1E-6
				(1.3E-5)				(6.5E-6)
$\lambda_{\Delta\sigma^2_W I(\mathcal{D})}$				-1.9E-5***				-7.8E-6
				(7.2E-6)				(6.4E-6)
λ_{SMB}		0.0012				0.0007		
		(0.0013)				(0.0013)		
λ_{HML}		0.0020*				0.0002		
		(0.0011)				(0.0010)		
λ_{WML}		0.0050***				0.0189***		
		(0.0015)				(0.0014)		
SSE	0.0015	0.0005	0.0009	0.0007	0.0041	0.0021	0.0033	0.0027

The Table presents results of Fama-MacBeth regressions. **The base assets are portfolios.** For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months - $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. The row labelled "SSE" presents the average sum of squared pricing errors for the given model.

Table 8: Fama-Macbeth regressions with different definitions for the disappointing event

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
a	0	0	0.5	0.5	1	1	2	2	3	3
b	0	-0.005	0	-0.005	0	-0.005	0	-0.005	0	-0.005
% of $\mathcal{D}_t$	45.67	21.36	44.40	23.36	42.14	24.60	38.80	25.69	36.76	26.25
Cons	0.0018	0.0018	0.0016	0.0017	0.0016	0.0017	0.0017	0.0017	0.0017	0.0018
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
λ_W	0.0065***	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***	0.0065***	0.0064***	0.0065***	0.0065***
	(0.0017)	(0.0017)	(0.0017)	(0.0016)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)
$\lambda_{\mathcal{D}}$	-0.3317***	-0.4413***	-0.4291***	-0.5686***	-0.4727***	-0.5519***	-0.4802***	-0.5092***	-0.4672***	-0.4852***
	(0.0746)	(0.0780)	(0.0850)	(0.0925)	(0.0913)	(0.0947)	(0.0871)	(0.0918)	(0.0813)	(0.0883)
$\lambda_{W\mathcal{D}}$	0.0064***	0.0065***	0.0064***	0.0067***	0.0064***	0.0066***	0.0064***	0.0059***	0.0062***	0.0058***
	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0011)	(0.0010)	(0.0011)	(0.0010)
λ_X	-8.3E-6***	-8.1E-6***	-7.7E-6**	-7.5E-6**	-7.5E-6**	-7.5E-6**	-7.5E-6**	-7.3E-6**	-7.5E-6**	-7.4E-6**
	(3.0E-6)	(3.0E-6)	(3.1E-6)	(3.0E-6)	(3.1E-6)	(3.1E-6)	(3.0E-6)	(3.0E-6)	(3.0E-6)	(3.0E-6)
$\lambda_{X\mathcal{D}}$	-6.9E-6***	-4.9E-6***	-6.9E-6***	-6.6E-6***	-6.9E-6***	-6.8E-6***	-7.1E-6***	-7.4E-6***	-7.4E-6***	-7.5E-6***
	(1.4E-6)	(9.8E-7)	(2.4E-6)	(2.2E-6)	(2.3E-6)	(2.4E-6)	(2.6E-6)	(2.6E-6)	(2.6E-6)	(2.6E-6)
adj R^2	0.0625	0.0606	0.0638	0.0630	0.0646	0.0637	0.0645	0.0636	0.0643	0.0640

The Table presents results of Fama-MacBeth (1973) regressions. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months: $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. The last row reports adjusted R^2 of given the model.
Each column uses a different definition for the disappointing event $I(\mathcal{D}_t)$. The disappointing region is defined as

$$r_{W,t} - a\,\frac{\sigma_{r_W}}{\sigma_{\Delta\sigma^2_W}}\Delta\sigma^2_{W,t} < b\ .$$

The values of a and b vary throughout the different specifications. "% of $\mathcal{D}_t$" denotes the percentage of days when the disappointing event occurs in an average one-year period.

Table 9: Fama-Macbeth regressions with different measures of market volatility

Panel A: 1964/07-2010/12										
	VIX		RV		GARCH		EGARCH		GJR-GARCH	
Cons					0.0020		0.0018		0.0017	
					(0.0019)		(0.0018)		(0.0018)	
λ_W					0.0065***	0.0080***	0.0065***	0.0077***	0.0065***	0.0077***
					(0.0016)	(0.0021)	(0.0017)	(0.0020)	(0.0017)	(0.0020)
$\lambda_{\mathcal{D}}$					-0.3282***	-0.4411***	-0.3317***	-0.4267***	-0.3308***	-0.4259***
					(0.0730)	(0.0978)	(0.0746)	(0.0957)	(0.0744)	(0.0956)
$\lambda_{W\mathcal{D}}$					0.0065***	0.0068***	0.0064***	0.0066***	0.0064***	0.0067***
					(0.0013)	(0.0015)	(0.0012)	(0.0014)	(0.0013)	(0.0014)
λ_X					-7.9E-6***	-4.3E-6	-8.3E-6***	-5.8E-6	-9.2E-6***	-5.3E-6
					(1.6E-6)	(2.7E-6)	(3.0E-6)	(4.1E-6)	(3.2E-6)	(4.7E-6)
$\lambda_{X\mathcal{D}}$					-5.2E-6***	-4.8E-6***	-6.9E-6***	-6.9E-6***	-7.3E-6***	-7.2E-6***
					(6.2E-7)	(8.0E-7)	(1.4E-6)	(1.9E-6)	(1.3E-6)	(1.8E-6)
adj R^2					0.0595		0.0625		0.0620	

Panel B: 1987/01-2010/09										
	VIX		RV		GARCH		EGARCH		GJR-GARCH	
Cons	0.0033		0.0034		0.0030		0.0028		0.0028	
	(0.0028)		(0.0028)		(0.0028)		(0.0027)		(0.0027)	
λ_W	0.0066***	0.0083***	0.0067***	0.0085***	0.0068***	0.0085***	0.0068***	0.0082***	0.0068***	0.0083***
	(0.0024)	(0.0027)	(0.0025)	(0.0028)	(0.0026)	(0.0028)	(0.0025)	(0.0026)	(0.0025)	(0.0027)
$\lambda_{\mathcal{D}}$	-0.2656***	-0.4073***	-0.2655***	-0.4010***	-0.2709***	-0.3969***	-0.2697***	-0.3822***	-0.2662***	-0.3803***
	(0.0888)	(0.1082)	(0.0947)	(0.1146)	(0.0944)	(0.1100)	(0.0960)	(0.1081)	(0.0953)	(0.1075)
$\lambda_{W\mathcal{D}}$	0.0060***	0.0066***	0.0061***	0.0067***	0.0063***	0.0068***	0.0062***	0.0066***	0.0062***	0.0066***
	(0.0019)	(0.0020)	(0.0019)	(0.0021)	(0.0020)	(0.0021)	(0.0019)	(0.0020)	(0.0019)	(0.0020)
λ_X	-1.1E-4*	-1.3E-5	-4.5E-5*	-2.6E-5	-6.3E-6**	-1.0E-6	-8.0E-6	-4.5E-6	-8.2E-6	-1.8E-6
	(6.2E-5)	(7.9E-5)	(2.7E-5)	(3.3E-5)	(3.0E-6)	(5.0E-6)	(5.6E-6)	(7.2E-6)	(6.0E-6)	(8.6E-6)
$\lambda_{X\mathcal{D}}$	-1.6E-4**	-2.3E-5	-4.2E-5**	-3.0E-5	-4.1E-6***	-4.5E-6***	-6.1E-6***	-6.8E-6***	-6.2E-6***	-7.1E-6***
	(7.3E-5)	(8.7E-5)	(1.8E-5)	(2.1E-5)	(7.7E-7)	(9.6E-7)	(2.2E-6)	(2.6E-6)	(2.0E-6)	(2.4E-6)
adj R^2	0.0417		0.0401		0.0403		0.0461		0.0446	

The Table presents results of Fama-MacBeth (1973) regressions using different approaches to measure market volatility. Appendix B describes the different approaches. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months: $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags.

Table 10: Correlations between measures of market downside risk

	β_i^{DM}	$\beta_i^{DM(2)}$	β_i^{SV}	β_i^{RSV}	$\beta_i^{RSV(2)}$	MES_i	$RMES_i$	$RMES_i^{(2)}$	β_i^{DV}	$\beta_i^{DV(2)}$
β_i^{DM}	1.000									
$\beta_i^{DM(2)}$	0.993	1.000								
β_i^{SV}	0.821	0.820	1.000							
β_i^{RSV}	0.861	0.858	0.959	1.000						
β_i^{RSV}	0.857	0.864	0.956	0.995	1.000					
MES_i	0.429	0.433	0.858	0.761	0.760	1.000				
$RMES_i$	0.497	0.499	0.832	0.860	0.856	0.897	1.000			
$RMES_i^{(2)}$	0.498	0.502	0.832	0.859	0.860	0.896	0.998	1.000		
β_i^{DV}	-0.046	0.028	0.010	-0.009	0.044	0.057	0.028	0.043	1.000	
$\beta_i^{DV(2)}$	0.069	0.141	0.105	0.090	0.142	0.105	0.084	0.099	0.981	1.000

The above table shows the correlation matrix of several measures of downside market risk. At every month t, we calculate the cross-sectional correlation of the measures estimated using daily data from the previous one-year period. The values presented in these tables are the time-series averages of these cross-sectional correlations over the sample period. The sample period is July, 1963 - December, 2010. The following measures are presented in the Table:

$$\beta_i^{DM} = \frac{Cov\left[R^e_{i,t}, r_{W,t} \mid \mathcal{D}_t\right]}{Var\left[r_{W,t} \mid \mathcal{D}_t\right]}$$

$$\beta_i^{DM(2)} = \beta_{iW} + \beta_{iW\mathcal{D}} \qquad \text{(β-s from the GDA5 model)}$$

$$\beta_i^{SV} = \frac{E\left[R^e_{i,t} r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]}$$

$$\beta_i^{RSV} = \frac{E\left[R^e_{i,t} r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]} - \frac{E\left[r_{W,t} \mid \mathcal{D}_t\right]}{E\left[r^2_{W,t} \mid \mathcal{D}_t\right]} E\left[R^e_{i,t}\right]$$

$$\beta_i^{RSV(2)} = a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}} \qquad \text{(β-s from the GDA5 model and the a coefficients are given in (43))}$$

$$MES_i = E\left[-R^e_{i,t} \mid \mathcal{D}_t\right]$$

$$RMES_i = E\left[-R^e_{i,t} \mid \mathcal{D}_t\right] - E\left[-R^e_{i,t}\right]$$

$$RMES_i^{(2)} = a_W \beta_{iW} + a_{W\mathcal{D}} \beta_{iW\mathcal{D}} + a_{\mathcal{D}} \beta_{i\mathcal{D}} \qquad \text{(β-s from the GDA5 model and the a coefficients are given in (47))}$$

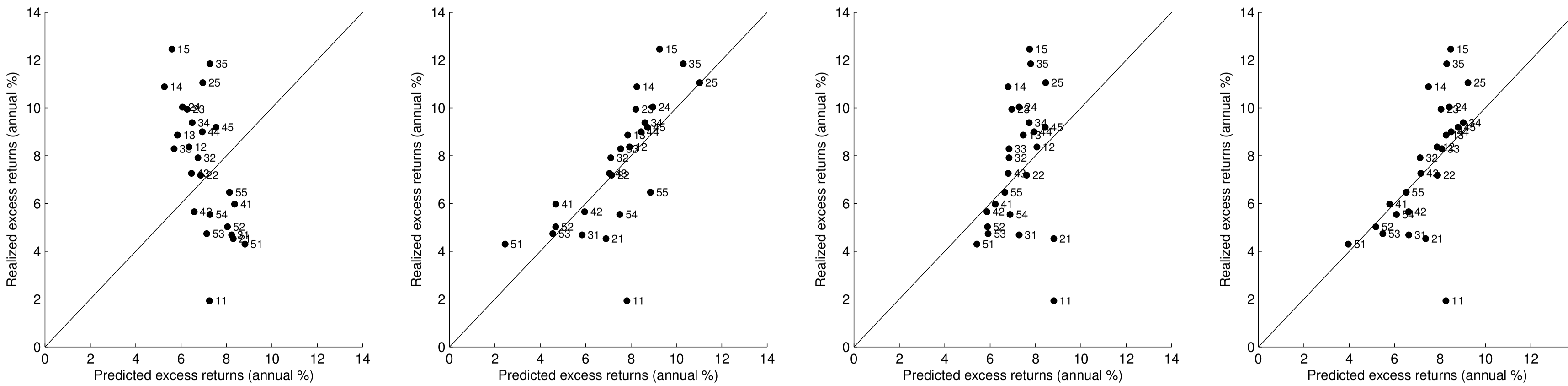
$$\beta_i^{DV} = \frac{Cov\left[R^e_{i,t}, \Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}{Var\left[\Delta\sigma^2_{W,t} \mid \mathcal{D}_t\right]}$$

$$\beta_i^{DV(2)} = \beta_{iX} + \beta_{iX\mathcal{D}} \qquad \text{(β-s from the GDA5 model)}$$

Figure 1: Actual versus predicted returns of portfolios

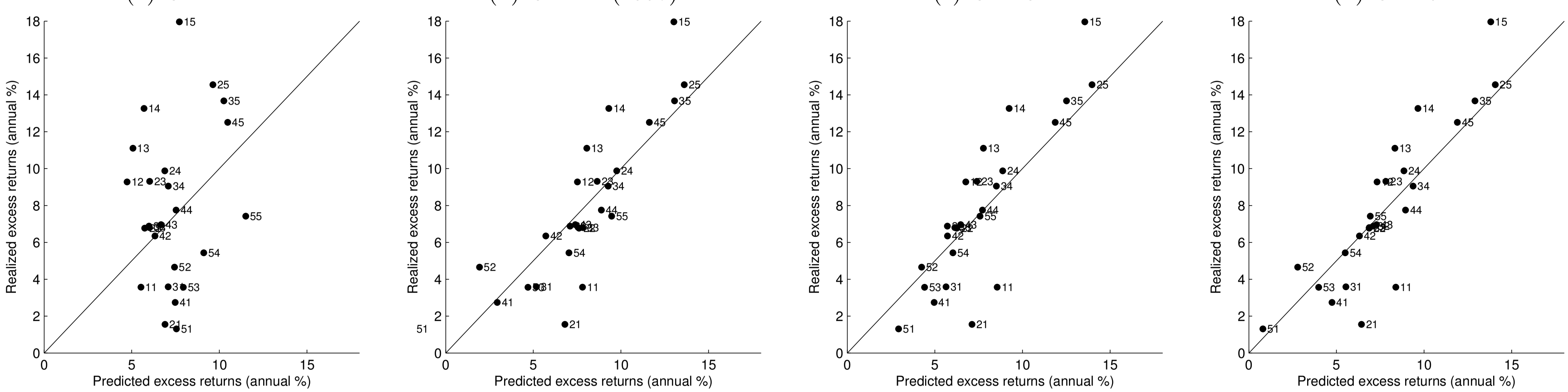
25 Size - Book-to-Market portfolios

(a) CAPM (b) Carhart(1997) (c) GDA3 (d) GDA5



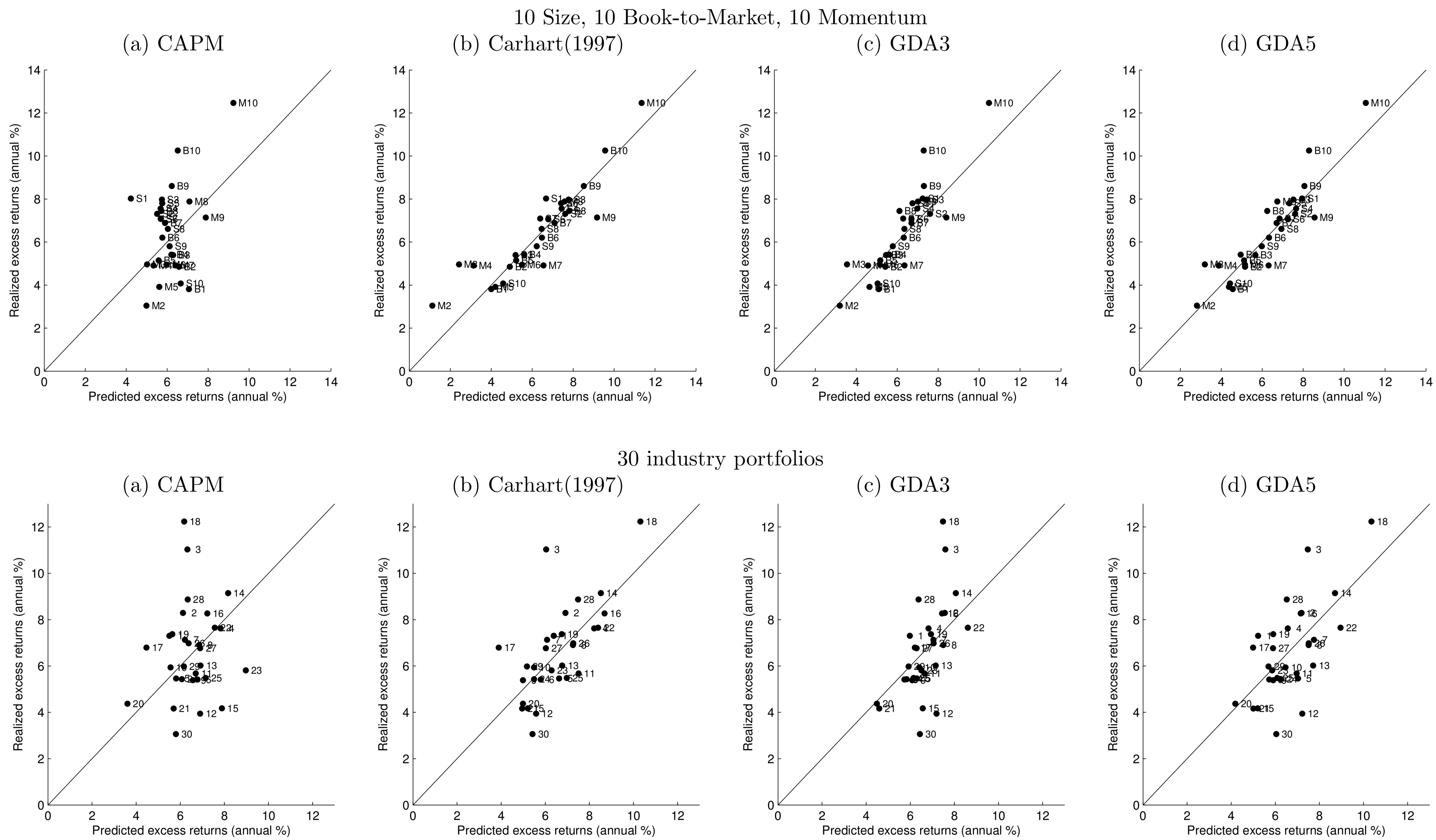
25 Size - Momentum portfolios

(a) CAPM (b) Carhart(1997) (c) GDA3 (d) GDA5



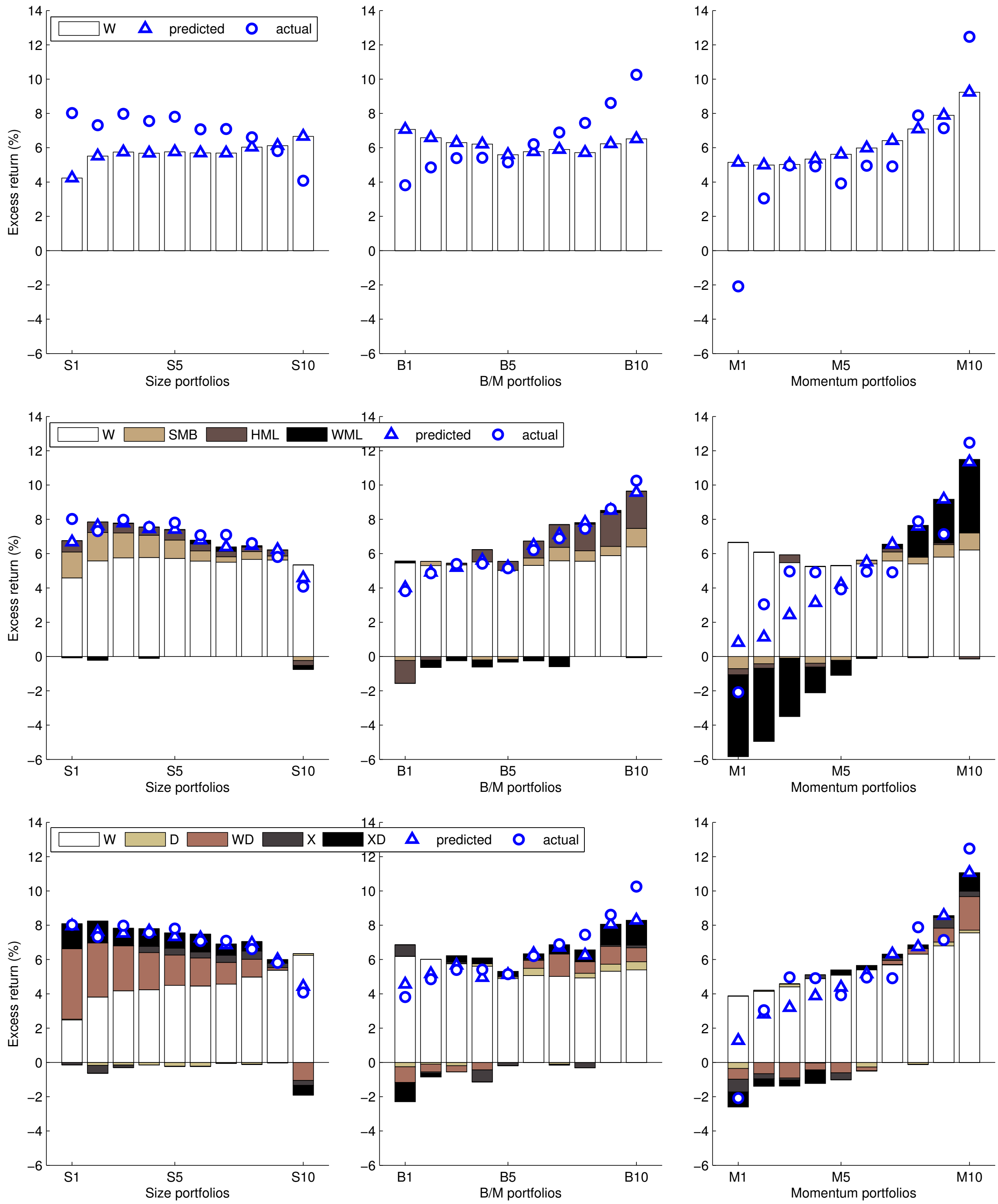
This figure shows the average excess returns for the 5×5 Size - B/M (top row, portfolio ij corresponds to the i-th size and j-th B/M quintile); and 5×5 Size - Momentum (bottom row, portfolio ij corresponds to the i-th size and j-th momentum quintile) portfolios against the average predicted returns from models reported in Table 7.

Figure 2: Actual versus predicted returns of portfolios



This figure shows the average excess returns for the 10 Size (labelled "S"), 10 B/M (labelled "B"), and 10 momentum (labelled "M") (top row); and 30 industry (bottom row) portfolios against the average predicted returns from models reported in Table 7.

Figure 3: Returns on the 10 Size, 10 Book-to-Market and 10 Momentum portfolios



This figure shows the decomposition of the average predicted excess return of 10 Size (left column), 10 B/M (middle column), and 10 momentum (right column) portfolios. Each part represents $E[\beta_{f_j} \cdot \lambda_{f_j}]$ connected to factor f_j from the standard CAPM (top row), the Carhart (1997) (middle row), and the GDA5 (bottom row) models. The corresponding $E[\lambda_{f_j}]$ estimates are those presented in the lower-left panel of Table 7. The symbol $\triangle$ represents average predicted return (sum of the parts), while $\circ$ represents actual average excess return of the portfolios.