Regional Stabilization Through Factor Mobility and Government Intervention

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Introduction

In this project I am interested in how labor and capital mobility enhances the economic integration of different regions. The focus is more on business cycle dynamics than on growth.
The main questions can be summarized the following way. Suppose there are two regions North and South. South is hit by a negative total factor productivity shock. What would be the response to this shock - would people move away from the South or would factories move to the relatively more lucrative North? Which effect would be more important quantitatively? What would happen if capital and labor is more costly to reallocate? How does the existence of federal redistribution change these results?
Although these questions are simple enough, it is difficult to find a framework that can answer them. The theoretical challenges are threefold. First, the workers when deciding where to move, must take into account their lifetime expected utility in all regions. Second, the spatial distribution of regions must be carefully modeled to be able to deliver a quantitative prediction. Third, if workers are allowed to both move and save/invest, then the distribution of different workers with different asset holding must be "remembered" across periods, generating a computationally demanding problem. Technically the economy will be similar to a Bewley-Huggett Aiyagari model with at least as many state variables as number of locations, making any business cycle analysis unfeasible.
In a broader perspective I would like to understand the fundamental difference between the economic integration of the United States and the European Union. Since the seminal paper of Mundell (1961), it has been the consensus that labor mobility across regions is essential for a currency union to be able to operate properly. And, while there is evidence that labor mobility is lower in Europe than in the US., it is unclear how important this difference is. It is a possibility that the generated welfare losses would not be much lower

in the EU if it would possess a US-level of internal migration or that higher capital mobility can counteract the lower labor mobility. Recent discussion about deepening economic integration in order to prevent serious recessions are indicating, that enhancing capital and labor mobility can reduce the impact of business cycle fluctuations - but so far it has been difficult to evaluate the effectiveness of these proposals absent a coherent framework.

Related Literature

There is an extensive literature related to the topic and the approach taken here is going to be a mixture of these. The most basic approach is more or less purely statistical as in Asdrubali et al. (1996) where the correlations of the data are exploited to quantify how the different channels contribute to interstate redistribution. Similarly Blanchard and Katz (1992) looks at employment data to conclude that migration is more important than job creation for regional stabilization. Exploring the data alone cannot answer the research questions for two reasons. First, there is an identification problems - for example, the effect of federal redistribution itself cannot be determined, as federal funds might be counter-cyclical by construction because of automatic stabilizers, and thus empirically one will find that they lead a decrease in output or consumption. Partial solution to this issue is focusing on "exogenous" increases in interstate redistribution by Romer and Romer (2014), but at the expense of omitting data. This implies that the use of a structural model is unavoidable. Moreover, previous empirical papers focused primarily on one channel of redistribution only - either fiscal intervention, labor or capital mobility and struggled to include them both in one framework. This, however is a more general problem in the literature and therefore I will discuss related papers separately for labor and for capital mobility and for federal redistribution.

Capital markets within the US seems to be behaving according to the neoclassical framework and are absent of the puzzles that are inherent to the international economics literature as documented by Kalemli-Ozcan et al. (2010). Therefore in my setup I will consider a capital market that is relatively friction-less - individuals will owns shares of an investor/capitalist firm that will reallocate the capital stock without any constraints. Of course, when I consider a counterfactual economy with higher capital mobility, I implement a costly capital reallocation.

Labor mobility in general is modeled in a static framework where a presence of some fixed factor in each region is assumed. The earliest generation of models built on the Rosen-Roback model as in Nieuwerburgh and Weill (2010) but this would be inadequate for business cycle analysis due to the somewhat involved computations. More recent advances by Redding (2016) in the theory of trade combines migration with an Eaton and Kortum (2002) model offering multiple advantages. These models inherit the property

of the existence of a unique equilibrium, easy comparative statics and a realistic, flexible depiction of the spatial dimension of the economy and therefore is widely adopted for quantitative work (see for example Ahlfeldt et al. (2012) or Zhu and Tombe (2015)). A dynamic version, where agents take into account the lifetime utility, not just one period gains as in Caliendo et al. (2015) is indeed going to form the core of the model adopted here. Farhi and Werning (2014) points out another important connection linking trade to migration as it redistributes wealth from the region that takes in migrants if the home bias motive is present, and thus trade is essential to include for yet another reason.
Federal redistribution is probably the most investigated and well understood topic from above. Usually a dynamic stochastic environment is used for assessing counter-factual policy change as in Beraja et al. (2015) or in Pennings (2016). The difficulty therefore is not going to be in this branch, but rather in uniting all these separate literature in one common framework.

Data

The two main data source for this project is the Bureau of Economic Analysis (BEA) and the American Community Survey (ACS). In general, the former is going to be used for aggregate variables, whereas the latter will be a useful source of micro data. The structural model is going to be estimated at the quarterly frequency, on data from 2000 to 2015. At this stage for computational reasons, I will assume that there are only two regions within the US - South, as defined by the Census "Southeastern" region and North, that incorporates the rest of the US. Yearly data at the regional level on migration rate of employed workers between the age of 18 and 65 is constructed from ACS, regional nominal output and wages (at this point, average wages) from BEA [1]. Quarterly data at the aggregate level for inflation, investment, interest income, nominal output is also downloaded from BEA. Note that regional variables are available at the yearly frequency, whereas aggregate variables are available at quarterly frequency and thus I have to use Kalman-Filter techniques to obtain the "missing" observations for the regional's, the idea being that the business cycle component of the US as a whole will be helpful determining the same for the regions within. In the baseline one sector model, the GDP deflator will be used to deflate both regions, nominal variables. In the multi-sector model, the price indexes will be constructed within the model. For the linearized solution, I will assume that the economy is approximated around the steady state (so South is not catching up to North) which implies that the data has to be filtered in line with DSGE traditions, using one-sided HP Filter.
To establish some facts about the internal migration within the US, I conduct reduced

[1]though for wages I intend to use the ACS data later on

form analysis on the ACS data and I do this at the state level, not according to the North-South division. I use the ACS to aggregate various income measures and create a variable that measures the percentage of people who moved to a particular state last year[2]. Only internal migration within US states will be considered. With this aggregation I obtain a panel data where each observation corresponds to a state-year combination. I find that regressing the percentage of newcomers on the wage rate is significant (see Table 1) - indicating that indeed, historically richer parts of the US attract more people. However, when I run a regression with the growth rate of wages as the main explanatory variable, I find that states with higher wage growth rates attract more people indicating that there is also the possibility of a business cycle channel, states that are performing better in a given period attract more people. Both of these findings are robust to various specifications[3]. It is worth mentioning that the only significant component determining the flow of migration is wage income - capital income, government redistribution plays no role (if anything, government redistribution decreases internal migration in line with previous findings in the literature). The fact that capital income plays little role does not mean that capital mobility is not important in determining labor flows - it only indicates that it must primarily operate through the wage rate - which, in the structural model, will be affected by the amount of capital available.

Another important feature of the data is the aggregate evolution of internal migration - it tends to be highly procyclical as illustrated by plotting the year fixed effects coefficients on Figure 1. This will be important in determining how to model the cost of moving. Theoretically, it can either be a utility- or a nominal cost. If the cost of moving is in terms of utility then the aggregate fluctuations will only change the aggregate migration levels if the cross sectional wage dispersion changes.

It is important to note that a large fraction of the variance of the dependent variable is unexplained, suggesting that the migration decision is primarily determined by non-economic factors, most likely idiosyncratic shocks like marriage, language, amenities (weather etc) but at the very least, economic variables do play a role. Any attempt at the understanding of the economics of migration should take this into account by allowing this unexplained factor enter the model.

[2]The ACS has population weights that are important in the construction of these measures - more details and the dataset itself can be accessed through `https://usa.ipums.org/usa/` (Genadek et al. (2015))

[3]Taking mean wages/total income, ignoring fixed effects etc.

Table 1: Regression Results

	(1) % of migrant population	(2) % of migrant population
logwage	0.0342	
	(8.24)	
D.logwage		0.0150
		(2.94)
Constant	-0.312	0.0280
	(-7.48)	(45.35)
Observations	816	765

t statistics in parentheses

Note: State and time fixed effects are also included. logwage is the median wage in each state and due to taking first differences, the year 2000 is lost.

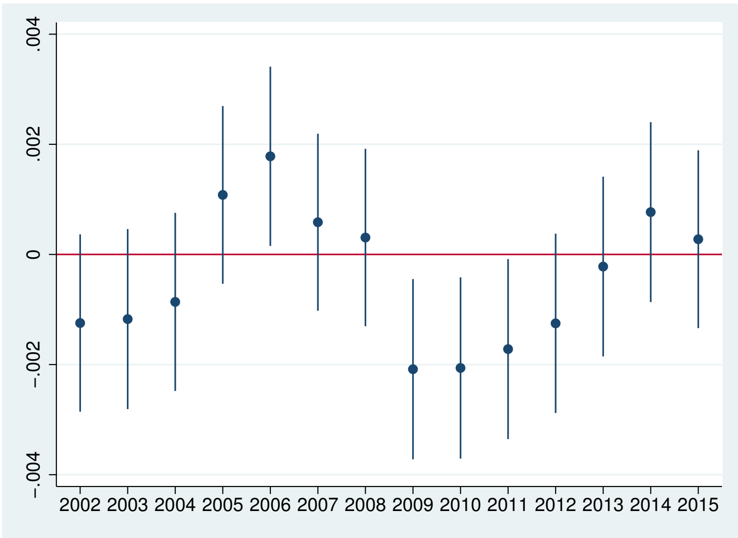


Figure 1: Time fixed effects for D.logwage

Model description

Time is discrete, denoted as t and a regional variable has an index $i = \{1 \dots N\}$. There are a continuum of workers of measure $\sum_i L_{i,t}$ living for an infinite amount of time, consuming in each period, unable to save, but able to choose which of the regions to live in, taking into account their lifetime utility. There is also a single capitalist firm who owns all the capital stock and only cares about maximizing their present discounted value W, where the flow profits are denoted as D_t. The regions differ in their capital stock K_{it}, population L_{it} and productivity of their respective representative firm Z_{it}. There are N goods, freely traded ,albeit subject to iceberg cost $d_{i,j}$ [4], across regions for consumption and investment purposes. ϕ_i share of the capitalist firm is owned by the current residents of region i, with $\sum_i \phi_i = 1$.

Worker's problem

Workers are free to move across the regions after they have consumed and worked this period. If they decide to migrate to location j they must pay a cost $p_{i,j,t}\tau_{i,j}$, as they must purchase $\tau_{i,j}$ goods from location j that they are not allowed to consume - think of a would-be immigrant purchasing furniture etc. in their new home. One might be worried that this will benefit the region that attracts the most worker, but in fact, due to that in calibration, migration will will be very low, these sunk costs will never have an important general equilibrium effect. Also, they draw an idiosyncratic taste shock each period for each location, $\epsilon_{i,t}$. This taste shock will determine how much they value staying in each location absent monetary considerations. It is entirely possible that more people would stay in the sunny South than in the cold North if the income of the workers would not be so much larger in the North and the taste preference is going to capture that. They consume their wage, $w_{i,t}$ and they also receive an equal share (conditional on location) of the dividend as transfers $tr_{i,t}$ from the capitalist firm. Denoting the aggregate state vector as $S_t = \{K_{i,t}, Z_{i,t}, L_{i,t}\}^i$ and the value function of a worker living in region i as $\tilde{V}_i(S_t, \{\epsilon_{j,t}\}^j)$ we can formulate their Bellman equation:

$$
\begin{aligned}
\tilde{V}_i(S_t, \{\epsilon_{j,t}\}^j) &= \max_{\{c_{i,l,t}\}^l, j\in\{1,2\}} u(\Gamma(\{c_{i,l,t}\}^l)) + \nu\epsilon_{jt} + \mathbb{E}\big[\beta\tilde{V}_j(S_{t+1}, \{\epsilon_{j,t}\}^j)\big] \\
\text{st. :} \sum_l p_{i,l,t}c_{i,l,t} &= w_{i,t} + tr_{i,t} - p_{i,j,t}\tau_{i,j} \\
\Gamma(\{c_{i,l,t}\}^l) &= \Big[\sum_l \gamma_l^{\frac{1-\sigma}{\sigma}} c_{i,l,t}^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}} \\
u(C) &= \frac{C^{1-\gamma}}{1-\gamma}
\end{aligned}
$$

where β is the discount factor, ν is the parameter controlling the variance of the idiosyncratic shocks,$c_{i,l,t}$ is the individual's consumption of good l, when he is located at region i at time t, for a price $p_{i,l,t}$, and the elasticity of substitution across goods is σ with weights γ_j, and across time it is γ. This problem can be solved in two stages. In the first stage we assume that a location decision has already been made so that j is fixed, and the consumer is just maximizing the consumption, conditional on their income. That is they

[4] In order to sell 1 unit of good j in region i, the producer must deliver $d_{i,j}$ unit, the rest "melts" away in transit

solve the static problem of:

$$\max_{\{c_{i,l,t}]^l} \Gamma(\{c_{i,l,t}\}^l)$$
$$\text{st. : } \sum_l p_{i,l,t} c_{i,l,t} = w_{i,t} + tr_{i,t} - p_{i,j,t}\tau_{i,j}$$

The solution to this first stage is:

$$c_{i,l,t} = \frac{(w_{i,t} + tr_{i,t} - p_{i,j,t}\tau_{i,j}) p_{i,l,t}^{-\sigma} \gamma_l^{1-\sigma}}{P_{i,t}^{1-\sigma}}$$
$$P_{i,t} = \Big[\sum_l (\gamma_l p_{i,l,t})^{1-\sigma}\Big]^{\frac{1}{1-\sigma}}$$
$$\Gamma(\{c_{i,l,t}\}^l) = \frac{w_{i,t} + tr_{i,t} - p_{i,j,t}\tau_{i,j}}{P_{i,t}}$$

where $P_{i,t}$ is the natural price index in region i. The second stage problem thus can be summarized as:

$$\tilde{V}_i(S_t, \{\epsilon_{j,t}\}^j) = \max_{j\in\{1,2\}} \mathbb{E}\big[\beta \tilde{V}_j(S_{t+1}, \{\epsilon_{j,t}\}^j) - \tilde{\tau}_{i,j} + \nu\epsilon_{jt}\big]$$
$$\text{where: } \tilde{\tau}_{i,j} = -\frac{(\frac{w_{i,t}+tr_{i,t}-p_{i,j,t}\tau_{i,j}}{P_{i,t}})^{1-\gamma}}{1-\gamma}$$

As I do not want to keep track of the whole distribution of $\epsilon_{i,t}$, I choose the distribution of ϵ_{it} as type 1 extreme value distribution, with cdf. $F(\epsilon) = exp(-exp(-\epsilon - \bar{\gamma}))$ with $\bar{\gamma} = \int_{-\infty}^{\infty} xexp(-x - exp(-x))dx$ and by doing so, I can reformulate the problem as

$$V_i(S_t) = \nu \log \sum_j \exp(\beta \mathbb{E}_{S_{t+1}}(V_j(S_{t+1})) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}}$$

as if a representative (or expected) consumer would allocate the labor. The main intuition is that the maximum of random variables are going to be distributed the same way if the underlying distribution is chosen wisely. This can be used to get a closed form for the population flows $\mu_{i,j}$ (from region i to region j):

$$\mu_{i,j}(S_t) = \frac{exp(\beta \mathbb{E}_{S_{t+1}} V_j(S_{t+1}) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}}}{\sum_l exp(\beta \mathbb{E}_{S_{t+1}} V_l(S_{t+1}) - \tilde{\tau}_{i,l})^{\frac{1}{\nu}}}$$

implying that the population evolves according to:

$$L_{i,t+1} = \sum_j \mu_{j,i} L_{j,t}$$

These derivations are presented in the appendix. Relative to Caliendo et al. (2015) the main change is that the environment here has aggregate shocks and that the the cost of migration is not in terms of a utility loss.

Final good producer

There is a good produced by each region's representative firm with Cobb-Douglas technology:

$$exp(Z_{it})K_{it}^{\theta}L_{it}^{1-\theta}$$

The first order necessary condition thus under the assumption of law of one price:

$$w_{i,t} = p_{i,i,t}(1-\theta)exp(Z_{it})(\frac{K_{it}}{L_{it}})^{\theta}$$
$$r_{i,t} = p_{i,i,t}\theta exp(Z_{it})(\frac{K_{it}}{L_{it}})^{\theta-1}$$

Capitalist

The capitalist firm owns all the capital that must be allocated in the respective regions and maximizes the present discounted value of profits. Denoting their value function as $W(S_t)$ they solve:

$$\begin{aligned} W(S_t) &= \max_{\{I_{it},K_{i,t+1},\{\omega_{i,j,t}\}^j\}^i} D_t + \beta\mathbb{E}\big[W(\{K_{i,t}, Z_{i,t}, L_{i,t}\}^i)\big] \\ \text{st.}\ :\ K_{i,t+1} &= \phi\Big(\frac{I_{it}}{K_{it}}\Big)K_{it} + (1-\delta)K_{it}\ \forall i \\ I_{i,t} &= \Gamma(\{\omega_{i,j,t}\}^j)\ \forall i \\ D_t &= \sum_i \big[r_{i,t}K_{it} - \sum_j (p_{i,j,t}\omega_{i,j,t})\big] \end{aligned}$$

with $\phi\Big(\frac{I_{it}}{K_{it}}\Big) = \Phi\cdot\Big(\frac{I_{it}}{K_{it}}\Big)^{1-\alpha}$. Note that the capitalist firm must buy the investment good ω locally in order to increase the capital stock and is subject to the same technology as the consumers are, and thus requires their problem to be solved in two stages. The first stage problem is the cost minimization for a given investment level:

$$\begin{aligned} &\min_{\{\omega_{i,j,t}\}^j} \sum_j p_{i,j,t}\omega_{i,j,t} \\ &\text{st.}\ :\ I_{i,t} = \Gamma(\{\omega_{i,j,t}\}^j) \end{aligned}$$

yielding the same price indexes with the total cost of investing $I_{i,t}$ equal to $P_{i,t}I_{i,t}$ and

$$\omega_{i,j,t} = P_{i,t}^{\sigma}I_{i,t}\gamma_j^{1-\sigma}p_{i,j,t}^{-\sigma}$$

Now the second stage, dynamic problem can be written as:

$$\begin{aligned} W(S_t) &= \max_{\{I_{it},K_{i,t+1}\}^i} D_t + \beta\mathbb{E}\big[W(\{K_{i,t}, Z_{i,t}, L_{i,t}\}^i)\big] \\ \text{st.}\ :\ K_{i,t+1} &= \phi\Big(\frac{I_{it}}{K_{it}}\Big)K_{it} + (1-\delta)K_{it}\ \forall i \\ D_t &= \sum_i \big[r_{i,t}K_{it} - P_{i,t}I_{i,t}\big] \end{aligned}$$

Taking FOC-s after substituting out $K_{i,t+1}$ and D_t gives:

$$I_{i,t}) \quad P_{i,t} = \Big(\frac{I_{i,t}}{K_{i,t}}\Big)^{-\alpha}(1-\alpha)\Phi\beta\mathbb{E}W_{K_{i,t+1}}$$

The Envelope condition is:

$$K_{i,t}) \quad W_{K_{i,t}} = r_{i,t} + \Big[\Big(\frac{I_{i,t}}{K_{i,t}}\Big)^{1-\alpha}\alpha\Phi + (1-\delta)\Big]\beta\mathbb{E}W_{K_{i,t+1}}$$

Substituting the expected derivative out:

$$W_{K_{i,t}} = r_{i,t} + \Big(\frac{I_{i,t}}{K_{i,t}}\Big)\frac{\alpha P_{i,t}}{1-\alpha} + \frac{(1-\delta)P_{i,t}}{(1-\alpha)\Phi}\Big(\frac{I_{i,t}}{K_{i,t}}\Big)^{\alpha}$$

and then substituting forward yields an Euler equation for each region:

$$(\frac{I_{it}}{K_{it}})^{\alpha} = \frac{\beta\Phi}{P_{i,t}}\mathbb{E}_t\Big[(1-\alpha)r_{i,t+1} + P_{i,t+1}(\alpha\frac{I_{i,t+1}}{K_{i,t+1}} + \frac{1-\delta}{\Phi}(\frac{I_{i,t+1}}{K_{i,t+1}})^{\alpha})\Big]$$

Equilibrium

Recursive Competitive Equilibrium with static regional ownership, prices $\{p_{i,j}, r_i, w_i\}$, decision rules $\{c_{i,l,j}\}$ (index j denotes the consumption conditional on leaving to region j), $\{K_i, L_i\}$, $\{\omega_{i,j}, K_i', I_i\}$, migration rates $\mu_{i,j}$ transfer rule $\{tr_i, T_i\}$, profits $\{D\}$, value functions $\{V_i\}, W$ such that:

- given prices, the decision rules solves the worker's, the firm's and the capitalist firm's problem
- Good markets clear:

$$\sum_{i,j} d_{i,l}(c_{i,l,j}\mu_{i,j}L_i + \omega_{i,l} + \tau_{i,l}\mu_{i,l}L_i) = exp(Z_l)L_{l,t}^{1-\theta}K_{i,t}^{\theta}$$

- Labor market clears
- Law of one price holds:

$$p_{i,j} = d_{i,j}p_j^j$$

- Capital market clears
- Transfer rule satisfies:

$$T_i = tr_i L_i$$

and

$$\sum T_i = D$$

- Fixed regional ownership of capital stock:

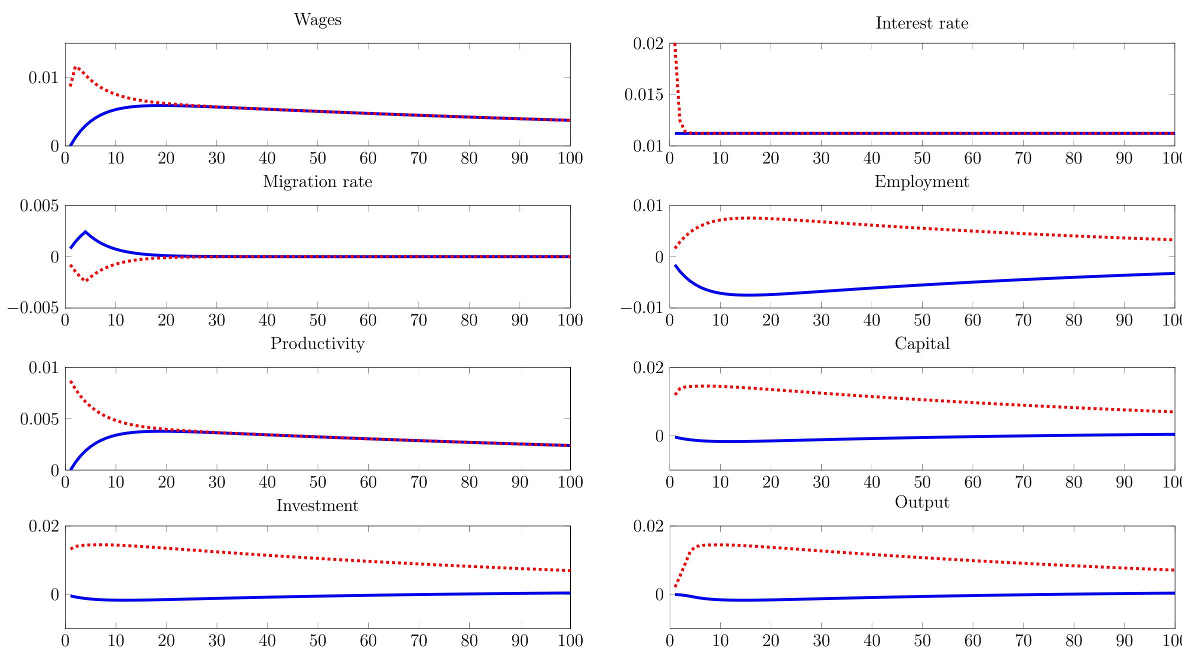
$$T_i = \phi_i D$$

A word of caution here is appropriate. A unique equilibrium is not guaranteed to exist and unlike in Caliendo et al. (2015) the exact parameter restrictions required are far more difficult to establish here. In general, if there is a unique equilibrium, then because the demand system exhibits gross substitution, it must be the case that there is a unique stable steady state. In the trade literature, the existence and uniqueness hinges on the fact that the congestion forces dominate agglomeration effect - that is, no region dominates the other regions such that it attracts all workers. The main component in congestion is the existence of a fixed factor: land, housing or capital. In this model, capital is not fixed so it acts as a congestion force within period - more workers deciding to go to a particular location will depress the wages - but has as an agglomeration effect across periods as if for whatever reason, more workers ended up in a particular location, more investment will happen there as the expected return will be higher. Therefore all variables affecting the dynamics of the capital stock (depreciation δ for example) are essential to pin down the equilibrium.
Moreover the productivity is treated as a variable and not as a parameter and as a consequence the cross-correlation and auto-correlation enters the considerations for the existence of the equilibrium. Intuitively, if total factor productivity is highly auto-correlated, then a lucky region will persistently attract investment and population and thus make the gains permanent.
The idiosyncratic taste shocks and the cost of moving end up playing the most important role of congestion. As long as there are workers who are willing to live in the poorer regions or are unwilling to afford to leave it, the equilibrium will exist. Unfortunately taste shocks are somewhat difficult to interpret. In some trade models they are isomorphic to labor productivity in each location (see Redding (2016) and Zhu and Tombe (2015)) but in this setup, the sake of aggregation requires that the shocks are independent. This is because the future continuation value in a particular region must be the same for all workers - this is why the taste shock of moving to a particular destination *next* period affects a worker *this* period. In reality, such region specific labor productivity is likely to be auto-correlated and thus I must refrain from using such an interpretation.

Calibration

The calibration follows Backus et al. (1994) for every parameter that is not related to migration or trade.There are two regions, $N = 2$, $i \in \{1, 2\}$ or {h ,f } or {South, North} - I will use these names for the regions interchangeably, such that North is referred to as foreign, or the second region. For the migration parameters ν and $\tau_{i,j} = \tau$ for $i \neq j$ and 0 otherwise, I target the gross labor flow of $\mu = 2\%$ in steady state. I set $\delta = 1$, that is, total depreciation - this simplifies calculation to ensure the existence of unique equilibrium. The trade cost $d_{i,j} = d = 2.5$ for $i \neq j$ and 0 otherwise , that is quite substantial and in line with the trade literature. For now, I am going to assume that in the steady state the two economies are of equal size in terms of population - I am unsure about which parameter (productivity or demand) should handle the different sizes To understand the main contribution of trade and the cost of moving I run a baseline model assuming only one sector, that the cost of moving is in terms of utility and that the capitalist firm just consumes it's profits. I calibrate the productivity process such that both in the baseline, one sector model and in the fully fledged multisector model the output volatility of the data is matched. The impulse responses to a positive foreign total factor productivity shock can be seen on Figures 2 and 3.

Figure 2: Impulse response functions for the one sector economy

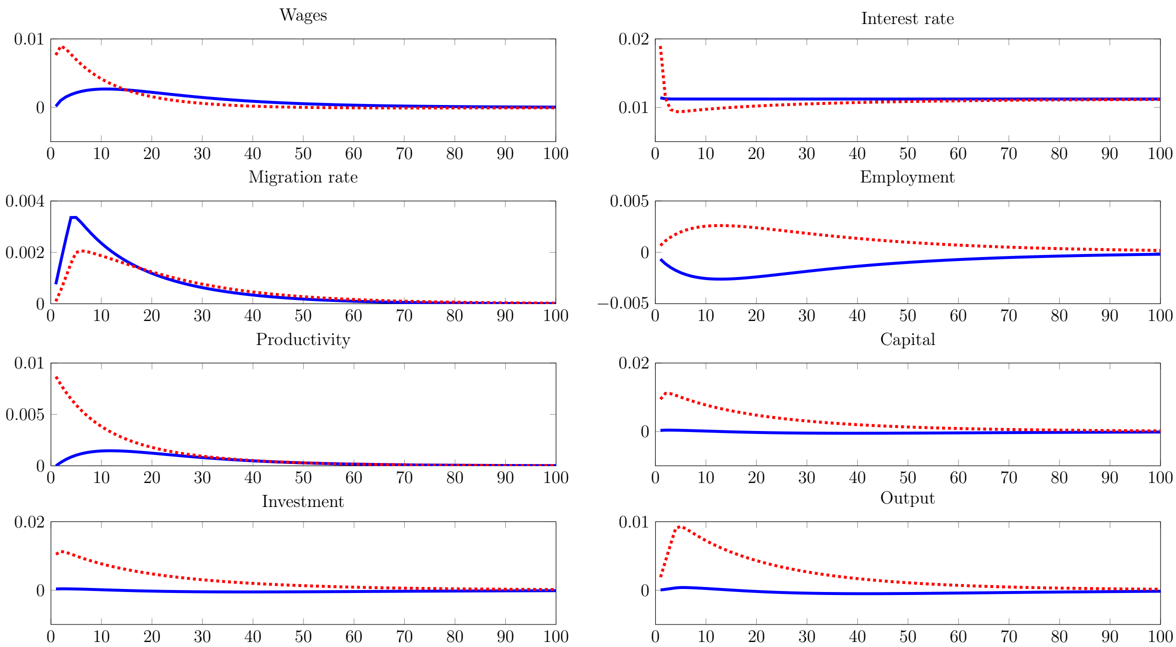


Foreign tfp shock, foreign variables are dotted

Results

The model is solved using Dynare, by linearizing around the steady state. There are two key failures of the baseline, one-sector model relative to the data. First, there is a positive spatial correlation between the migration rates in the data, but the model necessarily produces a negative correlation as can be understood from the impulse responses. A

Figure 3: Impulse response functions for the multi-sector economy



Foreign tfp shock, foreign variables are dotted

positive shock in the foreign economy implies that home is getting relatively poorer as the good produced by home is a perfect substitute to the foreign good and because the cost of moving is in utility terms, the absolute gains in overall production has no positive wealth effect on individuals decision to go to locations they prefer - hence migration from foreign decreases and from home it increases purely through the relative productivity channel. Absent positive spatial correlation of regional productivity, the perfect substitution of goods also implies a large negative spatial correlation of output in the model - whereas in the data it is a large positive correlation.

The statistics of the fully fledged multi-sector model economy are actually very close to the data even though the model has not been estimated yet as can be seen in Table 2. The correlation of output with migration and wages with migration is, however much higher in the model than in the data. The main reason for this is that the capitalist's problem is very much simplified: capital completely depreciates so there is very little income to redistribute, linear utility (and no stochastic discount factor inherited from the "owners"), no transaction cost, deep pockets (can sustain any losses in equity) and that the ownership of the capitalist firm is fixed. There is also no direct federal redistribution yet in the model. This all leads to the conclusion that wage income is overrated in the model and fluctuations in wage will be the primary reason of generating transitional migration.

The last experiment considers what would happen if the migration rate would decrease to $\mu = 1\%$ in steady state due to a permanent increase in the migration cost. This will lead to a less volatile ouptut (and not shown yet, but more volatile consumption). This happens because the two economies cannot respond as much to shocks due to the barriers and the more productive economy cannot attract as much labor and to make things worse, capital reallocation decreases too. Therefore there is an interaction of capital and labor mobility that leads to a lessened absorption of the regional shocks, a mechanism that I am will highlight later on.

Table 2: Summary statistics

Correlations	Data	One sector	Multi-sector	Lower migration
South Output vs Migration from South	0.13	-0.15	0.76	0.83
South Output vs Migration from North	0.05	0.15	0.85	0.91
Migration from South vs from North	0.96	-1	0.96	0.97
South Output vs North Output	0.74	0.25	0.51	0.66
Migration from South vs South wage	0.07	0.01	0.88	0.89
Migration from South vs North wage	-0.04	0.24	0.99	0.99
St. deviation (10^{-3})				
Migration rates	2	0.26	2	1.2
Output	5	5	5	4.7
Wage	6	4	5	5
Employment	2	2	1	0.7

Note: Data is the cycle component of the one sided HP filtered data apart for migration and interest rate

Conclusion - Next Steps

There are multiple objectives for what I would like to further achieve with this project. The motivation must be strengthened to show how internal migration and capital flows are related to the business cycle - with the possibility of not only looking at US data. I plan to modify the model itself in two points. In order to fit in more with business cycle models, I would like to add labor choice - this should be easily done as it would be a static decision due to the timing of events. As discussed, I would also like to interpret the taste shock as region specific labor productivity.

The capitalist's problem is too simplified to model the capital mobility realistically. Allowing the ownership of the capitalist firm to be dynamic would assign for a role to capital income redistribution. From the impulse response it can also be seen that the deep pocket assumption is restricting as only one interest rate reacts to shock so there is no reallocation going on. Government transfers will also reallocate labor income.

Allowing for more regions (or equivalently sectors) would be beneficial in fitting the data better and allowing for different counter-factual to be constructed. Doing so would increase the computational burden if I abandon the linearization and go for a global solution. The latter would be necessary if I choose to make the capitalist problem constrained.

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Appendix A - Aggregation

$$\tilde{V}_i(S_t, \epsilon_{1t}, \epsilon_{2t}) = \max_{j\in\{1,2\}} \mathbb{E}\big[\beta\tilde{V}_j(S_{t+1}, \epsilon_{1,t+1}, \epsilon_{2,t+1}) - \tilde{\tau}_{i,j} + \nu\epsilon_{jt}\big]$$

Note that expectations are always conditional on S_t and never on ϵ_t, and therefore to ease the notation, the conditionals are omitted. It is crucial that ϵ_t is iid for the calculations, but less so that S_t does not affect the distribution of ϵ_t.
Define $\Psi^i_t(S_t)$ as:

$$\Psi^i_t(S_t) = \mathbb{E}_{\epsilon_{j,t}} \max_j\{\nu\epsilon_{j,t} - \tilde{\tau}_{i,j} + \beta\mathbb{E}\tilde{V}_j(S_{t+1}, \epsilon_{1,t+1}, \epsilon_{2,t+1})\}$$

Denote $\bar{\epsilon}^{j,l}_t(S_t)$ as :

$$\bar{\epsilon}^{j,l}_t(S_t) = \frac{\beta(\hat{V}_j(S_t) - \hat{V}_l(S_t)) - (\tilde{\tau}_{i,j} - \tilde{\tau}_{i,l})}{\nu}$$

with $\hat{V}_j(S_t) = \mathbb{E}\tilde{V}_j(S_{t+1}, \epsilon_{1,t+1}, \epsilon_{2,t+1})$. Define $V_j(S_{t+1}) = \mathbb{E}_{\epsilon_{1,t+1},\epsilon_{2,t+1}}\tilde{V}_j(S_{t+1}, \epsilon_{1,t+1}, \epsilon_{2,t+1})$ implying that, due to the law iterated expectations. $\hat{V}_j(S_t) = \mathbb{E}_{S_{t+1}}V_j(S_{t+1})$.
Writing out the maximum and substituting in $\bar{\epsilon}$ yields:

$$\begin{aligned}
\Psi^i_t(S_t) &= \sum_j \int_{-\infty}^{\infty} (\beta\hat{V}_j(S_t) - \tilde{\tau}_{i,j} + \nu\epsilon_{j,t}) f(\epsilon_{j,t}) \prod_{l\neq j} F(\bar{\epsilon}^{j,l}_t(S_t) + \epsilon_{j,t}) d\epsilon_{j,t} \\
&= \sum_j \int_{-\infty}^{\infty} (\beta\hat{V}_j(S_t) - \tilde{\tau}_{i,j} + \nu\epsilon_{j,t}) e^{-\epsilon_{j,t}-\bar{\gamma}} \exp(-e^{-\epsilon_{j,t}-\bar{\gamma}} \sum_l \exp(\bar{\epsilon}^{j,l}_t(S_t))) d\epsilon_{j,t}
\end{aligned}$$

Let $\lambda_{j,t}(S_t)$ and $\xi_{j,t}$ be

$$\lambda_{j,t}(S_t) = \log\sum_l \exp(\bar{\epsilon}^{j,l}_t(S_t))$$

$$\xi_{j,t} = \epsilon_{j,t} + \bar{\gamma}$$

to simplify the $\Psi^i_t(S_t)$ to

$$\Psi^i_t(S_t) = \sum_j \int_{-\infty}^{\infty} (\beta\hat{V}_j(S_t) - \tilde{\tau}_{i,j} + \nu(\xi_{j,t} - \bar{\gamma})) e^{-\xi_{j,t} - \exp(-(\xi_{j,t} - \lambda_{j,t}(S_t)))} d\xi_{j,t}$$

with one last change of variables of $\tilde{y}_{j,t} = \xi_{j,t} - \lambda_{j,t}(S_t)$ we get:

$$\begin{aligned}
\Psi^i_t(S_t) &= \sum_j e^{-\lambda_{j,t}(S_t)} \big[(\beta\hat{V}_j(S_t) - \tilde{\tau}_{i,j} + \nu(\lambda_{j,t}(S_t) - \bar{\gamma})) \\
&\quad + \nu\int_{-\infty}^{\infty} \tilde{y}_{j,t} \exp(-\tilde{y}_{j,t} - e^{\tilde{y}_{j,t}}) d\tilde{y}_{j,t}\big] \\
&= \sum_j e^{-\lambda_{j,t}(S_t)} (\beta\hat{V}_j(S_{t+1}) - \tilde{\tau}_{i,j} + \nu\lambda_{j,t}(S_t))
\end{aligned}$$

using the definition of $\bar{\gamma}$. Now substitute out $\lambda_{j,t}(S_{t+1})$ and then $\bar{\epsilon}_t^{j,l}(S_{t+1})$:

$$\begin{aligned}\Psi_t^i(S_t) &= \sum_j \exp(-\log \sum_l e^{-\bar{\epsilon}_t^{j,l}(S_t)})(\beta \hat{V}_j(S_t) - \tilde{\tau}_{i,j} + \nu \log(\sum_l \exp(-\bar{\epsilon}_t^{j,l}(S_t)))) \\ &= \nu \big[log \sum_j \exp(\beta \hat{V}_j(S_t) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}} \big]\end{aligned}$$

This allows us to reformulate the Bellman equation in expected utility (for the idiosyncratic shock):

$$V_i(S_t) = \nu \log \sum_j \exp(\beta \mathbb{E}_{S_{t+1}}(V_j(S_{t+1})) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}}$$

as we still have to take expectations with respect to S_{t+1}. Due to the law of large number, the probability that the continuation value of a particular region is larger than anywhere else is equal to the realized labor flow to that region. That is, define $\mu_{i,j}(S_{t+1})$ as the gross migration rate, the percentage of people going from region i to region j). Then it must hold that:

$$\begin{aligned}\mu_{i,j}(S_t) &= Pr_{\epsilon_t}(\frac{\beta \hat{V}_j(S_t) - \tilde{\tau}_{i,j}}{\nu} + \epsilon_{j,t} \geq \max_{l \neq j}\{\frac{\beta \hat{V}_l(S_t) - \tilde{\tau}_{i,l}}{\nu} + \epsilon_{l,t}\}) \\ &= \frac{\exp(\beta \hat{V}_j(S_t) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}}}{\sum_l \exp(\beta \hat{V}_l(S_t) - \tilde{\tau}_{i,l})^{\frac{1}{\nu}}}\end{aligned}$$

through very similar calculations as for the value function. Therefore migration rate $\mu_{i,j}(S_t)$ in terms of $V_i(S_t)$ is equal to :

$$\mu_{i,j}(S_t) = \frac{exp(\beta \mathbb{E}_{S_{t+1}} V_j(S_{t+1}) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}}}{\sum_l exp(\beta \mathbb{E}_{S_{t+1}} V_l(S_{t+1}) - \tilde{\tau}_{i,l})^{\frac{1}{\nu}}}$$

Appendix B - Summary of the equilibrium conditions

The following system of equations summarize the dynamic equilibrium that is solved through linearization:

$$
\begin{aligned}
V_i(S_t) &= \nu \log \sum_j \exp(\beta \mathbb{E}_{S_{t+1}}(V_j(S_{t+1})) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}} \\
\mu_{i,j}(S_t) &= \frac{exp(\beta \mathbb{E}_{S_{t+1}} V_j(S_{t+1}) - \tilde{\tau}_{i,j})^{\frac{1}{\nu}}}{\sum_l exp(\beta \mathbb{E}_{S_{t+1}} V_l(S_{t+1}) - \tilde{\tau}_{i,l})^{\frac{1}{\nu}}} \\
c_{i,l,j} &= \frac{(w_i + tr_i - p_{i,j}\tau_{i,j}) p_{i,l}^{-\sigma} \gamma_l^{1-\sigma}}{P_i^{1-\sigma}} \\
P_i &= \Big[\sum_l (\gamma_l p_{i,l})^{1-\sigma} \Big]^{\frac{1}{1-\sigma}} \\
\tilde{\tau}_{i,j} &= -\frac{(\frac{w_i + tr_i - p_{i,j}\tau_{i,j}}{P_i})^{1-\gamma}}{1-\gamma} \\
L_i &= \sum_j \mu_{j,i} L_j \\
w_i &= p_{i,i}(1-\theta) exp(Z_i) (\frac{K_i}{L_i})^{\theta} \\
r_i &= p_{i,i}\theta exp(Z_i) (\frac{K_i}{L_i})^{\theta-1} \\
\omega_{i,j} &= P_i^{\sigma} I_i \gamma_j^{1-\sigma} p_{i,j}^{-\sigma} \\
(\frac{I_i}{K_i})^{\alpha} &= \frac{\beta\Phi}{P_i} \mathbb{E}\Big[(1-\alpha) r_i' + P_i' (\alpha \frac{I_i'}{K_i'} + \frac{1-\delta}{\Phi} (\frac{I_i'}{K_i'})^{\alpha})\Big] \\
K_{i,t+1} &= \phi\Big(\frac{I_{it}}{K_{it}}\Big) K_{it} + (1-\delta) K_{it} \\
exp(Z_l) L_{l,t}^{1-\theta} K_{i,t}^{\theta} &= \sum_{i,j} d_{i,l}(c_{i,l,j}\mu_{i,j} L_i + i_{i,l} + \tau_{i,l}\mu_{i,l} L_i) \\
\frac{\phi_i D}{L_i} &= tr_i
\end{aligned}
$$