Information Sharing in Common Agency: When is Transparency Good?[*]

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Abstract

When should principals dealing with a common agent share their individual performance measures about the agent's unobservable effort for producing a public good? In a model with two principals who offer linear incentive schemes, we show that information sharing always increases total expected welfare if the principal who is less informed about the agent's effort also cares more about the agent's output. If the less informed principal cares somewhat (but not too much) less than the other principal about the agent's output, information sharing reduces total expected welfare. In our model the efficient information regime emerges as an equilibrium outcome.

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1 Introduction

In public economics, industrial organization, and corporate finance there are many instances in which a "team of principals" (e.g., citizens, suppliers, or shareholders) benefit from the output generated by an agent (e.g., public service provider, common marketing agent, or manager). For example, providers of public goods and services—such as research, education, and health care—affect the well being of the direct beneficiaries of the services as well as other members of society.[1] Even though output is a public good for the team of principals, the principals are often unable to centralize the provision of incentives to the agent.[2]

Agencies and joint ventures carrying out research in targeted areas (such as infectious diseases and alternative energy sources) often have a variety of differently informed sponsors, including private firms, non-profit organizations, and government programs.[3] In the context of education, stakeholders—including parents and local government authorities—have access to partial information about the performance of schools and teachers. Similarly, when contracting with health care providers (such as doctors and hospitals), government authorities, insurance companies, and employers observe different performance measures.

In recent years, an increasing amount of performance information has been made publicly available, in the form of report cards, provider profiles, consumer reports, and "league tables". The effect of public reporting of performance data is currently the subject of a heated debate in health care policy circles in the US; organizations such as the National Committee for Quality Assurance (NCQA), the National Quality Forum, and the Leapfrog Group are involved in public reporting of quality evaluation data.[4]

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[1]Better educated individuals are better citizens and infectious diseases are less likely to spread in a healthier population. By their very nature, public services typically affect multiple principals, as stressed by Tirole (1994) and Dixit (2002) among others. In their review of the empirical literature on the use of performance measures in the public sector, Propper and Wilson (2003) also discuss the multi-principal nature of public good provision.

[2]Similarly, shareholders and other corporate stakeholders share the fruits of the labor of managers, but have an incentive to free ride in providing costly incentives. We depart from Huddart (1993) by focusing on situations in which principals bear a private cost when providing incentives to the agent. See also Stiglitz (1985) for a discussion of common agency in the context of incentive provision in companies with diffuse ownership.

[3]For example, funding bodies for cancer research (such as Cancer Research UK, a private foundation, and the UK's publicly funded Medical Research Council) often co-finance research projects.

[4]Participation by health plans to public reporting organizations is typically voluntary. Some States have taken an active role in publishing report cards. A widely publicized example is New York State's publication of mortality rates for physicians and hospitals performing certain cardiac procedures. As

While most of the debate on information sharing has focused on privacy concerns or adverse reactions to narrowly defined performance measures, this paper contributes a novel argument for the potentially perverse effects of information sharing when multiple stakeholders have an incentive to free ride in their provision of incentives to a common agent.[5] We identify circumstances in which the stakeholders might be better off not sharing their performance measures to avoid exacerbating free riding in incentive provision.

If the principals were able to coordinate and provide centralized incentives, they would use all performance information available to design an incentive scheme that maximizes their total payoff. When instead incentive provision is decentralized, this second-best outcome (efficient under informational constraints) is typically unfeasible. Focusing on a third-best world with decentralized contracting, we ask: When should the principals commit to publicizing their respective performance information ("transparent contracting" regime)? When should they keep this information private and contract exclusively on their individual signal ("private contracting" regime)?

It is well known that when a monolithic principal bloc contracts with an agent, more information improves inference about the agent's effort, resulting in an unambiguous Pareto improvement.[6] When principals are decentralized, we identify a countervailing negative effect from additional information. Given that incentive schemes are strategic substitutes, more information encourages each principal to free ride on the incentives provided by other principals, causing an inefficient reduction in the effort induced.

We study this trade-off between information and free-riding in the context of a tractable common agency model in which two principals contract with a single agent under moral hazard. The agent's effort is unobservable and each principal observes a performance measure containing noisy information about effort. The two principals possibly differ in the quality of their information as well as in the fraction of the agent's output they obtain (or, equivalently, in the intensity of their preferences for output). We assume that the agent has constant absolute risk aversion preferences and quadratic effort cost, the performance

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suggested by David Dranove in private communication, information sharing among large employers is common in California (see the Pacific Business Group on Health, PBGH), but less so in the Midwest (see the Midwest Business Group on Health, MBGH).

[5]See Fung, Graham, and Weil (2007) for an extensive review of the costs and benefits of transparency policies.

[6]See Holmström (1979). More information is not necessarily optimal when the principal cannot commit ex-ante to an incentive scheme, as in career concern models. See Prat (2005) on the dark side of transparency in a model with implicit incentives.

signals are normally and independently distributed conditional on effort, and the incentive contracts are linear.[7]

By comparing outcomes resulting with transparent and private contracting, we find conditions for information sharing to increase or reduce welfare. We show that private contracting dominates transparent contracting if the less informed principal cares somewhat (but not too much) less than the other principal about the agent's output. Conversely, there is a large set of parameters in which transparent contracting dominates private contracting—in particular, this is the case when (i) the principal who is less informed about the agent's effort also cares more than the other principal about the agent's output and (ii) the principal who is less informed about the agent's effort cares much less than the other principal about the agent's output.

To understand the intuition for these results, consider the plight of a primitive family of *homo oeconomicus*. After the slaying of Abel by Cain, Adam and Eve (the principals) are determined to provide proper education for their third son Seth (the agent). In the baseline scenario, Adam and Eve each have access to different (and conditionally independent) performance measures and provide separate incentives to Seth. We posit that Adam's information is less accurate than Eve's information, and that Adam cares less than Eve about Seth's performance. Should Adam and Eve agree to share their information?

In the baseline case with private contracting the slope of the incentive contract offered by a principal does not depend on the slope offered by the other principal (see Proposition 1). When instead Adam and Eve share their information, the slopes of the incentive schemes are strategic substitutes for the principals. Because of this strategic effect, given that Eve provides more incentives than Adam, under transparent contracting Adam is induced to free ride on Eve's incentive provision. There is then a meaningful trade-off between inefficiency in the use of information shared by the principals and the overall increase in information at their disposal.

A necessary condition for private contracting to be more efficient than transparent contracting is that private contracting results in an increase in effort level, which happens when Adam cares sufficiently less than Eve about Seth's performance. However, this condition is not sufficient, as this increase in effort also carries a higher risk premium,

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[7]We borrow this model from Holmström and Milgrom (1987), who justify the optimality of linear incentive schemes for the case of a single principal in the context of a richer dynamic model. The restriction to linear incentive schemes is further discussed in Section 3 and footnote 13.

because incentives with private contracting are based on noisier information. Provided that Adam's relative share is not too small, the beneficial effect of the increase under private contracting dominates the savings in risk premium from transparent contracting.

Indeed, if Adam did not care at all about Seth's performance, he would provide no incentives whatsoever to Seth under private contracting. In this case, making Adam's information available to Eve clearly improves welfare. By continuity, this result also holds when Adam cares very little about Seth's performance. This is the intuition for our result that information sharing is socially beneficial when the less informed principal cares much less than the other principal about the agent's performance.

The paper proceeds as follows. Section 2 reviews the most closely related literature. Section 3 introduces our baseline common-agency model with two principals. Section 4 presents the case of private contracting, in which each principal observes her own performance measure. Section 5 analyzes the case of transparent contracting, in which both principals observe and contract on both performance measures. Section 6 compares outcomes and welfare levels achieved under private and transparent contracting. Section 7 analyzes the information regimes that arise in equilibrium when each principal decides voluntarily whether or not to disclose their information. Section 8 argues that our result also holds when principals offer a menu of contracts or agents are allowed to contract with a single principal. Section 9 concludes. The proofs of all the results are in the Appendix.

2   Literature

The theoretical analysis of common agency under moral hazard was spearheaded by Bernheim and Whinston (1986), who focus on the case in which the principals have possibly diverging objectives but access the same information. Our analysis of private contracting can be seen as an extension of this common agency framework that allows the principals to contract on different performance measures.

Holmström and Milgrom (1988) and Dixit (1996 and 1997) also analyze moral-hazard common agency models in which competing principals have access to different information, but in a multi-task environment.[8] In Holmström and Milgrom's (1988) model, the agent carries out two tasks and each principal cares about one of these two tasks.[9] In their

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[8] Holmström and Milgrom (1991) further analyze the multi-task model for the case of a single principal.

[9] Dixit (1996) and (1997) extend Holmström and Milgrom's (1988) analysis of common agency games

setting, Holmström and Milgrom (1988) briefly note that information sharing is clearly detrimental when the agent's tasks are technologically independent and the principals' performance measures are statistically independent. Given that under private contracting each principal has access to a performance measure about the task about which she cares, the common agency distortion is avoided altogether and the second-best outcome results. Hence, private contracting trivially dominates transparent contracting in their multi-task setting.

Departing from these previous models, our agent concentrates on a single task and the two principals care—to a possibly different extent—about the output resulting from this one task.[10] The single-task case is relevant for applications in which the output is a public good for the principals, as in the provision of health care, education, and research. In this setting, the outcome of common agency even under private contracting cannot be second-best. Hence, we find a non-trivial trade-off between the value of information and free riding in incentive provision.[11]

3 Model

An agent takes a one-dimensional effort on behalf of two principals. The cost of effort for the agent is $k\mu^2/2$, with $k > 0$. The agent receives payment w_i from principal i. The agent is risk averse with constant absolute risk-aversion (CARA) coefficient $r > 0$, implying utility function

$$U = 1 - e^{-r\left(w_1 + w_2 - \frac{k}{2}\mu^2\right)}$$

with reservation utility equal to zero.

There are two principals, $i = 1, 2$. In the baseline specification with private contracting, each principal observes a single noisy signal of the effort level chosen by the agent. If the

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with linear incentive schemes to the case with more than two principals, while keeping their multi-task formulation from which we depart.

[10]Being about the same task, the principals' performance measures cannot be statistically independent (as in the multi-task example discussed by Holmström and Milgrom 1988), though they can be independent *conditional* on the effort exerted by the agent, as in our model.

[11]With these notable exceptions, the literature on common agency with asymmetric information has focused mostly on hidden information models of adverse selection—see Martimort's (2007) survey and, in particular, Taylor (2004) and Calzolari and Pavan (2005) analyses of the incentives of principals to share information about customers in sequential screening models. Information sharing has been studied extensively in the context of oligopoly, where firms could share information about demand and/or cost (see, e.g., Raith 1996).

agent exerts effort μ, the performance measure observed by principal i is $x_i = \mu + \varepsilon_i$, where ε_i are independent normally distributed error terms with mean 0 and variances σ_i^2.

Private contracting is realistic in settings in which performance measures are privately observed by individual principals, given that courts have no power to compel third parties to disclose information. The fact that principal i can contract on x_i means that performance measure x_i is available to either principal i or the agent, and that a court can compel the knowledgeable party to reveal this information publicly. If, as we assume, x_i is only verifiable because principal i has some information that a court can compel, then principal j would be unable to contract on it. This is because in a contract dispute between principal j and the agent, the court would typically have no power to compel principal i (a third party) to testify or produce information.[12]

Each principal offers a wage schedule to the agent conditional on the performance measure observed. In particular, principal i offers wage schedule w_i to the agent. Principals are risk neutral with payoff functions

$$v^i = b_i x_i - w_i,$$

where $b_i > 0$ is principal i's benefit coefficient.

We assume that principals offer linear contracts. In the private contracting regime, the incentive scheme offered by principal i is

$$w_i(x_i) = \alpha_i x_i + \beta_i = \alpha_i(\mu + \varepsilon_i) + \beta_i,$$

where α_i is the "slope" of the incentive contract (a.k.a. the piece rate). The restriction to linear contracts is often made in the literature because of its analytical convenience. Adapting the analysis of Holmström and Milgrom (1987) to the case with multiple principals, Holmström and Milgrom (1988) show that a principal's best reply to a linear contract by a competing principal is also to offer a linear contract.[13] However, equilibria in non-linear contracts cannot be ruled out.

Due to the noise in the performance measure, the agent receives an uncertain wage for any choice of effort. It is convenient to carry out the analysis in terms of the certainty

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[12]If, instead, the agent were to know x_i, then principal j would be able to contract on x_i.

[13]Holmström and Milgrom (1987) analyze a dynamic model in which a single principal contracts repeatedly with a risk-averse agent with CARA preferences and additively separable effort cost. They show that the optimal dynamic incentive scheme can be computed as if the agent were choosing the mean of a normal distribution only once and the principal were restricted to offering a linear contract.

equivalent the agent obtains upon choosing a given level of effort.[14] In fact,

$$CE = \alpha_1 \mu + \alpha_2 \mu + \beta - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2), \tag{1}$$

where $\beta = \beta_1 + \beta_2$ is a convenient shortcut as β_1 and β_2 are not uniquely determined in equilibrium. The last term is the risk premium required by the agent for the uncertainty borne. Summing the payoffs of the principals and the agent, total welfare is

$$W = b_1 \mu + b_2 \mu - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2). \tag{2}$$

Following Bernheim and Whinston (1986), we model the interaction between the principals and the agent as a two-stage game. In the first stage, the principals simultaneously commit to a single wage schedule to the agent, with payments contingent on observed performance measures.[15] In the second stage, the agent chooses an effort level, taking the wage schedules offered by the principals as given.

The information regime is fixed at stage zero before contracts are signed and determines which performance measures are available to each principal. Under private contracting (analyzed in Section 4) principal i can contract on performance measure x_i, while under transparent contracting (analyzed in Section 5) both principal 1 and 2 can contract on both performance measures, x_1 and x_2. Section 6 compares the total welfare achieved in the two regimes. Section 7 addresses whether the outcome that results in highest total welfare is achieved in equilibrium when each principal in stage zero chooses independently to make their performance measure available to the other principal.[16]

4 Private Contracting

Consider the common agency game in which each principal only observes her own signal about the agent's effort. The equilibrium of this common agency game is a triplet including

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[14]By definition, the certainty equivalent is the certain payment that gives the agent the same expected utility obtained with the original gamble: $1 - e^{-rCE} = 1 - Ee^{-r\left(\alpha_1(\mu+\varepsilon_1)+\beta_1+\alpha_2(\mu+\varepsilon_2)+\beta_2-\frac{k}{2}\mu^2\right)}$.

[15]The restriction that principals use simple take-it-or-leave-it offers rather than menus of contracts may involve a loss of generality in some common-agency games (see Peters 2001, Martimort and Stole 2002, Calzolari and Pavan 2006). Section 8.1 shows that our results are robust to competition in menus of contracts.

[16]When total welfare is higher under transparent contracting, information sharing results in an increase in the total payoff of the principals, given that the agent is held down to the reservation utility. However, the total payoff is shared among the principals in an arbitrary way. To analyze equilibrium information sharing we need to make additional assumptions on how the principals split the total payoff.

the agent's effort level, and the two linear incentive schemes offered by the principals, such that: (i) the agent chooses the effort level to maximize his expected utility, taking the incentive schemes offered by the two principals as given, and (ii) each principal offers the incentive scheme that gives her the highest expected payoff, taking as given the incentive scheme provided by the other principal and the agent's optimal choice rule.[17]

To find the equilibrium of the game, we solve principal i's optimization problem, taking as given the incentive scheme provided by principal j:

$$\max_{\alpha_i, \beta_i} \left\{ (b_i - \alpha_i)\mu - \beta_i \right\}$$

$$s.t. \quad \mu = \arg\max_{\widetilde{\mu}} \left\{ (\alpha_1 + \alpha_2)\widetilde{\mu} + \beta_1 + \beta_2 - \frac{k}{2}\widetilde{\mu}^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2) \right\} \quad (3)$$

$$(\alpha_1 + \alpha_2)\mu + \beta_1 + \beta_2 - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2) \geq 0. \quad (4)$$

The first constraint imposes that μ is incentive compatible for the agent, while the second ensures participation by the agent. In this formulation, common agency is intrinsic because the agent is not allowed to contract with a single principal.[18]

As a first step for solving principal i's optimization program, we determine the agent's effort choice from the incentive compatibility constraint (3) as a function of the total slope of the incentive schemes,

$$\mu = \frac{\alpha_1 + \alpha_2}{k}. \quad (5)$$

Substituting the binding participation constraint (4) into the objective function, principal i's problem becomes

$$\max_{\alpha_i} \left\{ (b_i + \alpha_j)\mu - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2) + \beta_j \right\}$$

$$s.t. \quad \mu = \frac{\alpha_1 + \alpha_2}{k}. \quad (6)$$

The interaction between principal j and the agent affects principal i's optimization problem through both the incentive compatibility and the participation constraint. A higher α_j increases principal i's marginal (and total) cost of raising the incentive compatible effort level by varying α_i. Additionally, a change in α_j affects principal i's optimization problem

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[17] As explained in Section 3 and footnote 13, there might be other equilibria in which principals do not offer linear contracts.

[18] Section 8.2 extends our model to delegated common agency, in which the agent has the additional option of accepting the offer of one principal, while rejecting the offer of the other principal.

through the agent's participation constraint. The payment the agent receives for any given unit of effort is increasing in α_j; therefore, a higher α_j implies that principal i has a lower wage threshold to meet when supporting effort μ. This decrease of the marginal costs of each unit of effort for principal i is equivalent to an increase in her marginal benefit for any unit of μ.[19]

Overall, the interaction between principal j and the agent imposes two externalities on principal i's optimization problem. First, a positive externality arises as the slope of the incentives provided by principal j, α_j, decreases principal i's cost for every unit of effort. Second, there is a negative externality as α_j increases the cost of implementing additional units of effort by varying α_i.[20] Substituting the agent's optimal effort into the maximand in (6), each principal's first order condition is

$$\frac{b_i + \alpha_j}{k} - \frac{\alpha_i + \alpha_j}{k} - r\sigma_i(\alpha_i\sigma_i) = 0 \qquad i,j = 1,2 \ \ i \neq j. \tag{7}$$

The two externalities exactly offset each other in this model, because contracts are linear and effort cost is quadratic.

The following proposition characterizes the equilibrium outcome of our common agency game under private contracting:

Proposition 1 (Private Contracting) *The slope of the equilibrium incentive scheme offered by principal i with private contracting is*

$$\alpha_i^{PP} = \frac{b_i}{1 + rk\sigma_i^2}, \ \ i = 1,2. \tag{8}$$

The equilibrium effort level exerted by the agent is

$$\mu^{PP} = \frac{b_1 + b_2}{k} \frac{1 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2)} \tag{9}$$

and the agent's expected payoff is equal to zero in equilibrium. The total welfare in equilibrium is

$$W^{PP} = \frac{rk\left[b_2^2\sigma_1^2 + b_1^2\sigma_2^2 + 2b_1b_2\left(\sigma_1^2 + \sigma_2^2\right)\right] + (b_1 + b_2)^2}{2k\left(1 + rk\sigma_1^2\right)\left(1 + rk\sigma_2^2\right)}. \tag{10}$$

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[19]This formulation reflects Bernheim and Whinston's (1986, page 927) observation that: "a principal can always compose his offer in two steps: he first undoes the offers of the other principals, and then decides upon some aggregate offer".

[20]In the more general model in which the error terms in the performance measures of the principals are not independent (but are still normally distributed), there would be an additional externality that enters through the agent's participation constraint. In that case, a change in α_j would also affect the risk premium required by the agent for his uncertain payment stream through the correlation. The sign of this externality depends on the sign of the correlation coefficient.

5 Transparent Contracting

We now analyze the equilibrium outcome when each principal observes (and can contract on) both performance measures, x_1 and x_2. We denote the slope of the incentive scheme offered by principal i by α_{i1} and α_{i2}, where α_{i1} refers to performance measure x_1 and α_{i2} refers to performance measure x_2.

To derive the equilibrium incentive schemes and the equilibrium effort level, we solve for the principals' optimization problem. Principal 1's optimization problem is

$$\max_{\alpha_{11},\alpha_{12},\beta_1} \{b_1\mu - (\alpha_{11} + \alpha_{12})\mu - \beta_1\}$$

$$s.t. \quad \mu = \arg\max_{\widetilde{\mu}} \left\{ (\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22})\widetilde{\mu} + \beta_1 + \beta_2 - \frac{k\widetilde{\mu}^2}{2} - \right.$$

$$\left. - \frac{r[(\alpha_{11}+\alpha_{21})^2\sigma_1^2+(\alpha_{12}+\alpha_{22})^2\sigma_2^2]}{2} \right\} \tag{11}$$

$$(\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22})\mu + \beta_1 + \beta_2 - \frac{k\mu^2}{2} - \frac{r[(\alpha_{11}+\alpha_{21})^2\sigma_1^2+(\alpha_{12}+\alpha_{22})^2\sigma_2^2]}{2} \geq 0. \tag{12}$$

The first constraint imposes that μ is incentive compatible for the agent, while the second constraint guarantees that the agent participates.

As a first step toward solving principal 1's optimization problem, we determine the agent's effort choice from the incentive compatibility constraint. The optimal effort for the agent is

$$\mu = \frac{\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22}}{k}. \tag{13}$$

Using the agent's binding participation constraint to express β_1 and substituting it in principal 1's objective function, principal 1's optimization problem becomes

$$\max_{\alpha_{11},\alpha_{12}} \left\{ (b_1 - \alpha_{11} - \alpha_{12})\frac{\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22}}{k} + \frac{(\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22})^2}{2k} - \right.$$

$$\left. - \frac{r[(\alpha_{11} + \alpha_{21})^2\sigma_1^2 + (\alpha_{12} + \alpha_{22})^2\sigma_2^2]}{2} + \beta_2 \right\}. \tag{14}$$

Note that the slopes $(\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22})$ are not uniquely determined in equilibrium. The Appendix shows that this is the result of the fact that each of the four linear equations corresponding to the first order conditions are linearly dependent.[21] Despite this, the

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[21] Clearly, this is a consequence of the linear contracts offered by the principals and the quadratic effort function of the agent.

values of $\alpha_{11} + \alpha_{12}$ and $\alpha_{21} + \alpha_{22}$, as well as the values of $\alpha_1 = \alpha_{11} + \alpha_{21}$ and $\alpha_2 = \alpha_{12} + \alpha_{22}$ are uniquely determined in equilibrium.

The following proposition characterizes the equilibrium outcome of our common agency game under transparent contracting:

Proposition 2 (Transparent Contracting) *The slope corresponding to signal m ($m = 1, 2$) of the equilibrium incentives scheme offered by principal i cannot be uniquely determined in equilibrium. The slope of the equilibrium incentive scheme related to signal m under transparent contracting is*

$$\alpha_m^{TT} = \alpha_{1m}^{TT} + \alpha_{2m}^{TT} = \frac{(b_1 + b_2)\,\sigma_n^2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}, \quad m, n = 1, 2 \quad n \neq m \quad . \tag{15}$$

The equilibrium effort level exerted by the agent is

$$\mu^{TT} = \frac{b_1 + b_2}{k} \frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}, \tag{16}$$

and the agent's expected payoff is equal to zero in equilibrium. The total welfare in equilibrium is

$$W^{TT} = \frac{(b_1 + b_2)^2}{2k} \frac{(\sigma_1^2 + \sigma_2^2)(\sigma_1^2 + \sigma_2^2 + 3rk\sigma_1^2\sigma_2^2)}{(\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2)^2}. \tag{17}$$

6   Welfare Comparison

How do the incentives under transparent contracting compare with those under private contracting? How does total welfare compare? To answer these questions, we contrast the first order condition associated with the optimization problem of principal 1 under transparent contracting

$$\underbrace{\frac{b_1 + \alpha_{21} + \alpha_{22}}{k}}_{\text{marginal benefit}} - \underbrace{\frac{\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22}}{k}}_{\text{marginal cost of effort}} - \underbrace{r\sigma_1^2(\alpha_{11} + \alpha_{21})}_{\text{marginal cost of risk}} = 0,$$

with the corresponding the first order condition under private contracting

$$\underbrace{\frac{b_1 + \alpha_2}{k}}_{\text{marginal benefit}} - \underbrace{\frac{\alpha_1 + \alpha_2}{k}}_{\text{marginal cost of effort}} - \underbrace{r\sigma_1^2\alpha_1}_{\text{marginal cost of risk}} = 0.$$

Principal 1 provides incentives to the agent to exert effort through $\alpha_{11} + \alpha_{12}$ with transparent contracting, and through α_1 with private contracting. Therefore, the marginal

11

benefits and the marginal costs of effort in the two information regimes are the same. However, the marginal cost of risk is different in the two regimes. With transparent contracting, principal 1 can control two variables, α_{11} and α_{12}, to provide incentives to the agent. In turn, principal 2 also controls two variables, α_{21} and α_{22}, and so imposes an externality on principal 1 through both these variables. Overall, even though with transparent contracting principal 1 has one more variable to manage risk, she also incurs an externality from principal 2 from one more channel. As a result, one cannot conclude a priori which regime results in higher effort and overall welfare.

For our subsequent analysis, we stipulate without loss of generality that the first principal is the one who observes the noisier signal, that is $\sigma_1 > \sigma_2$. As (9) and (16) indicate, a decrease in b_1 combined with an equal increase in b_2 (so that their sum is kept constant) increases the value of μ^{PP} for $\sigma_1 > \sigma_2$ but leaves the value of μ^{TT} unaffected. Thus, the equilibrium effort level is symmetric in b_1 and b_2 with transparent contracting, whereas it is not symmetric in b_1 and b_2 with private contracting.

Intuitively, it can be seen that for $\sigma_1 > \sigma_2$ the agent can be more efficiently incentivized by principal 2. However, as we are in a non-cooperative setting, if principal 2 has a low private valuation, she will have little incentive to induce the agent to exert extra effort. Therefore, the equilibrium effort level chosen by the agent increases if we "transfer" one unit of private valuation from principal 1 to principal 2, or, alternatively, we decrease the ratio b_1/b_2 by keeping the sum $b_1 + b_2$ of private valuations constant. For simplicity, we fix the value of $b_1 + b_2$ at 1 in the rest of the paper.

The comparison of equilibrium effort and total welfare can be most effectively done by expressing all the equilibrium variables as functions of a, defined as the ratio b_1/b_2 of principal 1's and principal 2's private valuations. The equilibrium effort level under private contracting

$$\mu^{PP} = \frac{1}{k}\left(\frac{a}{1+a}\frac{1}{1+rk\sigma_1^2} + \frac{1}{1+a}\frac{1}{1+rk\sigma_2^2}\right)$$

is a weighted average of $1/\left(1+rk\sigma_1^2\right)$ and $1/\left(1+rk\sigma_2^2\right)$. Given that $\sigma_1 > \sigma_2$, we have that an increase in a or, equivalently, an increase in the weight $a/\left(1+a\right)$ decreases the value of μ^{PP}. From (16), the equilibrium effort level under transparent contracting is independent of a. We show that there exists a threshold level a_0 for the ratio $a = b_1/b_2$ such that a higher effort level can be implemented under private contracting than under transparent contracting if $a < a_0$:

Lemma 1 (Effort Comparison) *Assume that $\sigma_1 > \sigma_2$. Then the equilibrium effort level is higher under private than transparent contracting, $\mu^{PP} > \mu^{TT}$, whenever $b_1/b_2 < a_0$, where*

$$a_0 := \frac{\sigma_2^2}{\sigma_1^2} \frac{1 + rk\sigma_1^2}{1 + rk\sigma_2^2} < 1. \tag{18}$$

Given that $a_0 < 1$ for $\sigma_1 > \sigma_2$, the equilibrium effort level is higher under transparent than private contracting if the two principals have the same private valuation, $b_1 = b_2$. For a numerical example, Figure 1.a plots the equilibrium effort levels as a function of a under both information regimes. (If instead $\sigma_1 < \sigma_2$, we have $\mu^{PP} > \mu^{TT}$ whenever $b_1/b_2 > a_0 > 1$.)

<center>FIGURE 1 ABOUT HERE</center>

We now seek conditions for total welfare to be higher under private contracting, so that more information is detrimental from the social point of view. For the purpose of comparing welfare under the two information regimes we decompose it into two components,

$$W = (B - C) + RP,$$

where the first "production" component represents joint expected benefits net of effort cost

$$B - C = (b_1 + b_2)\,\mu - \frac{k}{2}\mu^2$$

and the second "risk sharing" component represents the cost of the risk premium the agent demands

$$RP = -\frac{r}{2}(\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2).$$

Under transparent contracting, both the production and the risk sharing components are constant functions of a. It can be shown that $\mu^{PP} > \mu^{TT}$ implies $(B - C)^{PP} > (B - C)^{TT}$ whenever $\mu^{PP} < \mu^{FB}$. The following lemma summarizes our comparison of the production effects under the two information regimes (see also Figure 1.b for a numerical illustration):

Lemma 2 (Production Effect) *Assume that $\sigma_1 > \sigma_2$. If $b_1/b_2 < a_0$ as defined in (18), then total equilibrium benefits to the two principals net of the cost of effort is higher with private contracting than with transparent contracting: $(B - C)^{PP} > (B - C)^{TT}$.*

<center>13</center>

Comparing the risk sharing effects of the two information regimes is less straightforward. It can be shown that the relative magnitude of the risk premium under private and transparent contracting depends not only on the ratio a of principals' benefits, but also on the extent of the information difference $(\sigma_1 - \sigma_2)$ between them. The following lemma derives conditions under which the equilibrium risk premium for private contracting is larger than under transparent contracting:

Lemma 3 (Risk Sharing Effect) *Assume that $\sigma_1 > \sigma_2$. Then the equilibrium risk premium with private contracting RP^{PP} is larger than the equilibrium risk premium with transparent contracting RP^{TT} in the following three cases: (1) $\sigma_1 < \overline{\sigma}_1$ and $a_0 < b_1/b_2 < a_0^{RP}$; (2) $\sigma_1 = \overline{\sigma}_1$ and $a_0 < b_1/b_2$; and (3) $\sigma_1 > \overline{\sigma}_1$ and $a_0^{RP} < b_1/b_2 < a_0$ where explicit expressions for $a_0, \overline{\sigma}_1,$ and a_0^{RP} are reported in (18), (33), and (34).*

Figure 1.c illustrates this comparison when $\sigma_1 < \overline{\sigma}_1$ for the same numerical example used above. Now we can compare total welfare under the two information regimes (see also Figure 1.d):

Proposition 3 (Welfare Comparison) *Assume that principal 1 has a noisier signal than principal 2 $(\sigma_1 > \sigma_2)$. Then the total equilibrium welfare with private contracting is higher than with transparent contracting, $W^{PP} > W^{TT}$, when the principal 1 cares somewhat (but not too much) less than principal 2 about the agent's performance:*

$$\frac{b_1}{b_2} \in \left(a_0^W, a_0\right) \tag{19}$$

where explicit expressions for a_0^W and a_0 are reported in (36) and (18).

We conclude that a necessary condition for transparency to be socially undesirable is that the less-informed principal cares less about the agent's effort than the other principal: $b_1 < b_2$ and $\sigma_1 > \sigma_2$. Intuitively, private contracting can result in a higher effort level because free-riding is less of a factor in incentive provision. Indeed, the equilibrium effort under private contracting is higher than under transparent contracting for $b_1/b_2 < a_0$. Given that effort is under-provided in equilibrium (both under transparent and private contracting) compared to the second best, this higher level of effort, μ, is closer to the second best level. This higher effort level is socially beneficial because it results in increased benefits net of effort cost. This is because the production effect component of the total

14

welfare function is a quadratic function of the effort level (with the quadratic term having a negative coefficient) and therefore, it increases in μ when the effort level is below the first-best level (for $\mu < (b_1 + b_2)/k$). The increase in effort is also socially costly because of the additional risk imposed on the agent.

7 Equilibrium Information Regime

So far we have analyzed the effect of information sharing on social welfare, which is equal to the sum of the expected payoffs of the two principals. It is natural to wonder what would happen if the principals were to decide non-cooperatively whether or not to share their information, before contracting with the agent. This section analyzes the information regimes that arise in equilibrium of the pre-contractual game of information sharing in which each principal decides simultaneously whether or not to disclose her information to the other principal. The key question we address is whether the socially efficient outcome always emerges in equilibrium.

To analyze the incentives of each principal to unilaterally reveal her signal to the other principal, we first need to analyze the asymmetric scenario in which only one principal agrees to make her signal publicly available. Section 7.1 characterizes the equilibrium under such one-sided transparent contracting, while Section 7.2 analyzes the information regimes that can emerge as equilibrium outcomes.

7.1 One-Sided Transparent Contracting

In this section, we derive the equilibrium outcome in the common agency game in which principal 1 contracts only on her own performance measure, while principal 2 contracts on both performance measures. Such one-sided transparent contracting arises when principal 1 shares her performance measures with principal 2, while principal 2 does not share her performance measure with principal 1. In this information regime, principal 1 offers incentive scheme (α_1, β_1), while principal 2 offers incentive scheme $(\alpha_{21}, \alpha_{22}, \beta_2)$.

With these two incentive schemes, the agent's optimal choice of effort is equal to

$$\mu = \frac{\alpha_1 + \alpha_{21} + \alpha_{22}}{k}. \tag{20}$$

Principal 1's optimization problem is

$$\max_{\alpha_1,\beta_1} \{(b_1 - \alpha_1)\mu - \beta_1\}$$

$$s.t. \quad \mu = \frac{\alpha_1 + \alpha_{21} + \alpha_{22}}{k} \tag{21}$$

$$(\alpha_1 + \alpha_{21} + \alpha_{22})\mu + \beta_1 + \beta_2 - \frac{k}{2}\mu^2 - \frac{1}{2}r[(\alpha_1 + \alpha_{21})^2\sigma_1^2 + \alpha_{22}^2\sigma_2^2] \geq 0. \tag{22}$$

Substituting (20), this problem becomes

$$\max_{\alpha_1} \left\{(b_1 - \alpha_1)\frac{\alpha_1+\alpha_{21}+\alpha_{22}}{k} + \frac{(\alpha_1+\alpha_{21}+\alpha_{22})^2}{2k} - \frac{r[(\alpha_1+\alpha_{21})^2\sigma_1^2+\alpha_{22}^2\sigma_2^2]}{2} + \beta_2\right\}.$$

Similarly, principal 2's optimization problem is

$$\max_{\alpha_{11},\alpha_{12}} \left\{(b_1 - \alpha_{21} - \alpha_{22})\frac{\alpha_1+\alpha_{21}+\alpha_{22}}{k} + \frac{(\alpha_1+\alpha_{21}+\alpha_{22})^2}{2k} - \frac{r[(\alpha_1+\alpha_{21})^2\sigma_1^2+\alpha_{22}^2\sigma_2^2]}{2} + \beta_1\right\}.$$

By solving these optimization problems, we find the slope of the equilibrium incentive schemes:

Proposition 4 (One-Sided Transparent Contracting) *In the common agency game in which only principal 1 shares her signal, the slopes of the equilibrium incentive schemes are*

$$\alpha_1^{TP} = \frac{(\sigma_1^2 + \sigma_2^2 + rk\sigma_1^2\sigma_2^2)b_1 - rk\sigma_1^2\sigma_2^2 b_2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}, \tag{23}$$

$$\alpha_{21}^{TP} = \frac{\sigma_2^2(1 + rk\sigma_1^2)b_2 - \sigma_1^2(1 + rk\sigma_2^2)b_1}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}, \tag{24}$$

$$\alpha_{22}^{TP} = \frac{\sigma_1^2(b_1 + b_2)}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}, \tag{25}$$

and the equilibrium effort level is the same as under transparent contracting, $\mu^{TP} = \mu^{TT}$, as reported in (16). The agent's equilibrium expected payoff is equal to zero and the total welfare is the same as under transparent contracting, $W^{TP} = W^{TT}$, as reported in (17).

This completes our analysis of the regime TP, in which principal 1 shares her information but principal 2 does not. Information regime PT, in which principal 2 shares her information and principal 1 does not, can be analyzed exactly in the same way.

16

7.2 Equilibrium and Efficiency

Does the efficient information regime always arise in equilibrium when each principal independently decides whether to share or not her information? A preliminary step to address this question is to characterize the allocation of total surplus between the two principals in our different information scenarios: private, transparent, and one-sided transparent contracting. As seen above, the allocation of total surplus between the principals is not determined in equilibrium of this intrinsic common agency game.[22] Thus, it is only possible to endogenize the decision of information publication by making additional assumptions on how the principals share the total surplus they create in the different information regimes. Here, we proceed under the natural assumption that each principal receives a constant fraction of the total surplus.[23] We denote by γ the fraction of the surplus allocated to principal 1.

The information regime is determined according to the following game of information sharing. Each principal decides simultaneously whether to share her performance signal with the other principal. The payoffs of the principals are as follows:

	S	NS
S	$\gamma W^{TT}, (1-\gamma)W^{TT}$	$\gamma W^{TP}, (1-\gamma)W^{TP}$
NS	$\gamma W^{PT}, (1-\gamma)W^{PT}$	$\gamma W^{PP}, (1-\gamma)W^{PP}$

where W^{PP} and W^{TT} are defined by (10) and (17). By Proposition 4, this normal form representation of the signal disclosure game simplifies to:

	S	NS
S	W^{TT}, W^{TT}	W^{TT}, W^{TT}
NS	W^{TT}, W^{TT}	W^{PP}, W^{PP}

As can be seen from this representation, (S, S), (NS, S), (S, NS) are equilibrium outcomes whenever $W^{TT} > W^{PP}$ and (NS, NS) is the only equilibrium outcome whenever $W^{PP} > W^{TT}$. This means that no disclosure of signals is an equilibrium if and only if welfare in the private contracting case is higher than welfare in the transparent (and one-sided transparent) contracting case. This implies that the efficient outcome is always implemented in equilibrium:

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[22] As shown in Section 8.2, individual payoffs are also not determined in the symmetric equilibrium of the delegated common agency game with private contracting.

[23] If we instead allow the fraction of the total surplus allocated to a principal to depend on the information regime, we can clearly support any regime as an equilibrium outcome.

Proposition 5 (Efficiency of Equilibrium) *In the intrinsic common agency game the efficient signal disclosure pattern is always implemented in equilibrium.*

8 Robustness and Extensions

This section presents some robustness checks and extensions.

8.1 Competition in Menu of Contracts

We now discuss the role of the assumption that each principal offers a simple take-it-or-leave-it contract, rather than a menu including multiple contract offers. As shown in recent theoretical contributions on common agency (see footnote 15), a larger set of equilibria can often be supported with menus of contracts.

First, does our equilibrium survive when principals can offer menus? Applying Peters' (2003) Theorem 1 to our environment, we conclude that the equilibrium is robust to menus of contracts.

Second, is the set of equilibria enlarged by allowing for menus of contracts? The second part of Peters' (2003) no externality condition is not satisfied by moral hazard models of common agency, because a risk-averse agent's ranking of payoffs distributions offered by one principal depends on the distributions offered by the other principal. However, when the agent has CARA preferences, as in our model, the no externality assumption holds, as shown by Peters (2003) on page 104.[24] Hence, his Theorem 4 guarantees that no new equilibrium payoffs can be generated by allowing the principals to offer menus of contracts. We conclude that our results also hold when principals are allowed to offer menus of contracts.

8.2 Delegated Common Agency

Our baseline model assumes that the agent has the option of contracting with both principals or none. Alternatively, we could have allowed the agent to reject the offer of a principal, while accepting the contract offered by the other principal. We now briefly report on the robustness of our results for this alternative formulation with delegated

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[24] See also Attar, Piaser and Porteiro (2007) and Peters (2007).

common agency, investigated in detail in an earlier draft of this paper.[25]

For the case of private contracting, we find that the delegated common agency game has a symmetric equilibrium that is identical to the equilibrium of the intrinsic common agency game. In this equilibrium, the fixed terms (the βs) of the linear contracts are not uniquely determined. In addition, the delegated common agency game has two asymmetric equilibria, and in each of these equilibria the individual payoffs of the principals are uniquely determined. With transparent contracting, the equilibrium we characterize is again identical to the one resulting under intrinsic common agency. In this equilibrium, the payoffs of the individual principals are uniquely determined.

Finally, a similar trade-off to the one identified in the intrinsic common agency game (and also valid for the symmetric equilibria under delegated common agency) also arises when comparing the total welfare levels achieved in the asymmetric equilibria with private contracting and the equilibrium with transparent contracting. We conclude that our results also hold qualitatively under delegated common agency.

9   Conclusion

This paper analyzes information sharing in a common agency framework. We identify a trade-off between the value of information in agency and the distortions induced by increased free riding among multiple principals. We find that information sharing is beneficial for a large region of parameters. We characterize instances in which information sharing is detrimental. This happens in a (realistic) scenario in which the principal who is less interested in the agent's output has a less informative signal. However, the region of parameters in which information sharing is socially detrimental is relatively small.

We have developed these results in the context of a tractable but special model. While we believe that our main insights are robust to small deviations from our assumptions, we leave the analysis of more general environments to future work. This model remains tractable when there are more than two principals and the signals are conditionally correlated. Another natural extension of our model would allow the principals to acquire costly information. However, the analysis of this extension would depend critically on how the principals share their overall payoff.

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[25]For comparisons of intrinsic and delegated common agency in adverse selection (rather than moral hazard) environments see Calzolari and Scarpa (2008) and Martimort and Stole (2007).

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Appendix: Proofs

Proof of Proposition 1: We obtain the expressions for α_i from the first order conditions (7). Substituting α_1 and α_2 into (5), we find the equilibrium effort level (9). Welfare is then equal to

$$W^{PP} = \frac{(b_1 + b_2)^2}{k} \frac{1 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2)}$$

$$-\frac{1}{2}\frac{(b_1 + b_2)^2}{k}\frac{\left[1 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)\right]^2}{(1 + rk\sigma_1^2)^2(1 + rk\sigma_2^2)^2}$$

$$-\frac{r}{2}\frac{b_1^2\sigma_1^2(1 + rk\sigma_2^2)^2 + b_2^2\sigma_2^2(1 + rk\sigma_1^2)^2}{(1 + rk\sigma_1^2)^2(1 + rk\sigma_2^2)^2},$$

which boils down to (10).

Proof of Proposition 2: The FOCs for principal 1's problem (14) are

$$\alpha_{11} \;\; : \;\; -\frac{\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22}}{k} + \frac{b_1-\alpha_{11}-\alpha_{12}}{k} + \frac{\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22}}{k} - r(\alpha_{11} + \alpha_{21})\sigma_1^2 = 0, \quad (26)$$

$$\alpha_{12} \;\; : \;\; -\frac{\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22}}{k} + \frac{b_1-\alpha_{11}-\alpha_{12}}{k} + \frac{\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22}}{k} - r(\alpha_{12} + \alpha_{22})\sigma_2^2 = 0, \quad (27)$$

or more simply

$$b_1 - \alpha_{11} - \alpha_{12} - rk(\alpha_{11} + \alpha_{21})\sigma_1^2 \;\; = \;\; 0,$$

$$b_1 - \alpha_{11} - \alpha_{12} - rk(\alpha_{12} + \alpha_{22})\sigma_2^2 \;\; = \;\; 0.$$

Principal 2 has two similar FOCs with respect to α_{21} and α_{22}. Any of these four first order conditions can be obtained as a linear combination of the other three equations, with the coefficient vector $(1, -1, 1)$. Hence, the values of $\alpha_{11}, \alpha_{12}, \alpha_{21}$ and α_{22} are not determined uniquely.

The best response functions for principal 1 are

$$\alpha_{11} \;\; = \;\; \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2+rk(\sigma_1^2\sigma_2^2)}b_1 - \frac{\sigma_1^2+rk\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}\alpha_{21} + \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}\alpha_{22},$$

$$\alpha_{12} \;\; = \;\; \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}b_1 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}\alpha_{21} - \frac{\sigma_2^2+rk\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}\alpha_{22}.$$

Summing these two equations, we obtain principal 1's aggregate best reply function

$$\alpha_{11} + \alpha_{12} = \frac{\sigma_1^2+\sigma_2^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}b_1 - \frac{rk\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2+rk\sigma_1^2\sigma_2^2}(\alpha_{21} + \alpha_{22}), \qquad (28)$$

as a function of $\alpha_{21} + \alpha_{22}$. Solving the system of aggregate best replies for the two principals, we find the following equilibrium values

$$\alpha_{11} + \alpha_{12} = \frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}b_1 + \frac{rk\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}(b_1 - b_2), \qquad (29)$$

$$\alpha_{21} + \alpha_{22} = \frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}b_2 - \frac{rk\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2 + 2rk\sigma_1^2\sigma_2^2}(b_1 - b_2). \qquad (30)$$

Equations (26) and (27) only differ in their last term. By equating these last terms, we have

$$\frac{\alpha_{11} + \alpha_{21}}{\alpha_{12} + \alpha_{22}} = \frac{\sigma_2^2}{\sigma_1^2}. \qquad (31)$$

Summing equations (29) and (30) and adding the numerator to the denominator on both sides of the equation (31) we obtain the value of $\alpha_1^{TT} = \alpha_{11} + \alpha_{21}$ in Proposition (2). The value of $\alpha_2^{TT} = \alpha_{12} + \alpha_{22}$ can be obtained in a similar way.

The equilibrium effort level μ^{TT} can be obtained by substituting α_1 and α_2 into equation (13), while the equilibrium value for welfare can be obtained by a sequence of substitutions in the welfare formula developed for transparent contracting based on equation (2).

Proof of Lemma 1: Using expressions (9) and (16), $\mu^{PP} > \mu^{TT}$ is equivalent to

$$(\sigma_1 - \sigma_2)\left[\sigma_2^2\left(1 + rk\sigma_1^2\right) - \frac{b_1}{b_2}\sigma_1^2\left(1 + rk\sigma_2^2\right)\right] > 0.$$

As $\sigma_1 > \sigma_2$, this condition holds for $a = b_1/b_2 < a_0$ where $a_0 < 1$ is defined in (18).

Proof of Lemma 2: As $(B - C)(\mu)$ is a quadratic function of μ, $\mu^{PP} > \mu^{TT}$ implies $(B - C)^{PP} > (B - C)^{TT}$ whenever $\mu^{PP} < \mu^{FB}$. It can be shown that for $a = b_1/b_2$, $\mu^{PP} < \mu^{FB}$ whenever

$$\frac{1 + rk(\frac{1}{1+a}\sigma_1^2 + \frac{a}{1+a}\sigma_2^2)}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2)} < 1$$

or equivalently, whenever

$$a > -\frac{\sigma_2^2\left(1 + rk\sigma_1^2\right)}{\sigma_1^2\left(1 + rk\sigma_2^2\right)}$$

This is always true, as $a = b_1/b_2 > 0$.

Proof of Lemma 3: Let us first write RP^{PP} and RP^{TT} as a function of $a = b_1/b_2$ for $b_1 + b_2 = 1$. The risk premium for private contracting is

$$RP^{PP} = -\frac{r}{2}\left[\left(\frac{a}{1+a}\right)^2\frac{\sigma_1^2}{(1 + rk\sigma_1^2)^2} + \left(\frac{1}{1+a}\right)^2\frac{\sigma_2^2}{(1 + rk\sigma_2^2)^2}\right],$$

24

and under transparent contracting it is

$$RP^{TT} = -\frac{r}{2}\frac{\sigma_1^2\sigma_2^2(\sigma_1^2+\sigma_2^2)}{(\sigma_1^2+\sigma_2^2+2rk\sigma_1^2\sigma_2^2)^2},$$

which is unaffected by a.

Defining $a = b_1/b_2$, we have $RP^{PP} \gtreqless RP^{TT}$ if and only if

$$\left[\frac{\sigma_1^2}{(1+rk\sigma_1^2)^2} - \frac{\sigma_1^2\sigma_2^2\left(\sigma_1^2+\sigma_2^2\right)}{(\sigma_1^2+\sigma_2^2+2rk\sigma_1^2\sigma_2^2)^2}\right]a^2 - \frac{2\sigma_1^2\sigma_2^2\left(\sigma_1^2+\sigma_2^2\right)}{(\sigma_1^2+\sigma_2^2+2rk\sigma_1^2\sigma_2^2)^2}a$$
$$+\frac{\sigma_2^2}{(1+rk\sigma_2^2)^2} - \frac{\sigma_1^2\sigma_2^2\left(\sigma_1^2+\sigma_2^2\right)}{(\sigma_1^2+\sigma_2^2+2rk\sigma_1^2\sigma_2^2)^2} \lesseqgtr 0, \tag{32}$$

where the coefficient in front of a^2 is positive for

$$\sigma_1^2 + \sigma_2^2 + kr\sigma_2^2\left[2\sigma_1^2 + 2\sigma_2^2 + 2kr\sigma_1^2\sigma_2^2 - kr\sigma_1^2\left(\sigma_1^2-\sigma_2^2\right)\right] > 0,$$

which holds for $\sigma_1^2 < \overline{\sigma}_1^2$, where

$$\overline{\sigma}_1 := \sqrt{\frac{1 + kr\sigma_2^2\left(2+3kr\sigma_2^2\right) + \left(1+kr\sigma_2^2\right)\sqrt{1+9k^2r^2\sigma_2^4+2kr\sigma_2^2}}{2k^2r^2\sigma_2^2}}. \tag{33}$$

The two roots of the quadratic (32) are a_0 as defined in (18) and

$$a_0^{RP} := \frac{\sigma_2^2}{\sigma_1^2}\frac{kr\sigma_1^2+1}{kr\sigma_2^2+1}\frac{\sigma_1^2+\sigma_2^2+kr\sigma_1^2\left(2\sigma_1^2+2\sigma_2^2+2kr\sigma_1^2\sigma_2^2+kr\sigma_2^2\left(\sigma_1^2-\sigma_2^2\right)\right)}{\sigma_1^2+\sigma_2^2+kr\sigma_2^2\left(2\sigma_1^2+2\sigma_2^2+2kr\sigma_1^2\sigma_2^2-kr\sigma_1^2\left(\sigma_1^2-\sigma_2^2\right)\right)}. \tag{34}$$

Thus, if $\sigma_1 < \overline{\sigma}_1$ we have $RP^{PP} \geq RP^{TT}$ whenever $a_0 \leq b_1/b_2 \leq a_0^{RP}$. If instead $\sigma_1 > \overline{\sigma}_1$, we have $RP^{PP} \geq RP^{TT}$ whenever $a_0^{RP} \leq b_1/b_2 \leq a_0$.

Proof of Proposition 3: We establish that $W^{PP} > W^{TT}$ in the interval (19). Defining $A = kr\sigma_1^2$, $B = kr\sigma_2^2$, $C = 2kr\left(\sigma_1^2+\sigma_2^2\right)$, and

$$D = \frac{\left(\sigma_1^2+\sigma_2^2+3kr\sigma_1^2\sigma_2^2\right)\left(\sigma_1^2+\sigma_2^2\right)\left(1+kr\sigma_1^2\right)\left(1+kr\sigma_2^2\right)}{\left(\sigma_1^2+\sigma_2^2+2kr\sigma_1^2\sigma_2^2\right)^2},$$

$W^{PP} = W^{TT}$ is equivalent to $Ab_2^2 + Bb_1^2 + Cb_1b_2 + (b_1+b_2)^2 - D(b_1+b_2)^2 = 0$, which can be rewritten as the quadratic

$$(B-D+1)\left(\frac{b_1}{b_2}\right)^2 + b_2\left(C-2D+2\right)\frac{b_1}{b_2} + (A-D+1). \tag{35}$$

The discriminant $\Delta = (C-2D+2)^2 - 4(B-D+1)(A-D+1)$ is a perfect square, with square root

$$\frac{\left(\sigma_1^2-\sigma_2^2\right)2k^2r^2\sigma_1^2\sigma_2^2}{2kr\sigma_1^2\sigma_2^2+\sigma_1^2+\sigma_2^2}.$$

25

The two roots of the quadratic equation (35) are

$$a_0^W := \frac{\sigma_2^2 \left(\sigma_1^2 + \sigma_2^2 + 3kr\sigma_1^2\sigma_2^2 - kr\sigma_1^4\right)}{\sigma_1^2 \left(\sigma_1^2 + \sigma_2^2 + 3kr\sigma_1^2\sigma_2^2 - kr\sigma_2^4\right)} \tag{36}$$

and a_0, as defined in (18).

Under our assumption that $\sigma_1^2 > \sigma_2^2$, we have $a_0 < 1$. In addition, we have $a_0^W < a_0$, because this inequality is equivalent to $2kr\left(\sigma_1^2 - \sigma_2^2\right)\left(2kr\sigma_1^2\sigma_2^2 + \sigma_1^2 + \sigma_2^2\right) > 0$, given that the denominator of a_0^W is positive.

Proof of Proposition 4: The FOC for α_1 is

$$\frac{1}{k}\left(b_1 - \alpha_1\right) - r\sigma_1^2(\alpha_1 + \alpha_{21}) = 0, \tag{37}$$

whereas the FOCs for α_{21} and α_{22} are

$$\frac{1}{k}\left(b_2 - \alpha_{21} - \alpha_{22}\right) - r\sigma_1^2(\alpha_1 + \alpha_{21}) = 0, \tag{38}$$

$$\frac{1}{k}\left(b_2 - \alpha_{21} - \alpha_{22}\right) - r\sigma_2^2\alpha_{22} = 0. \tag{39}$$

From equations (37) and (38), we have

$$\alpha_1 = b_1 - b_2 + \alpha_{21} + \alpha_{22}. \tag{40}$$

Substituting (40) into (38) and solving for α_{21}, we obtain

$$\alpha_{21} = \frac{b_2 + kr\sigma_1^2(b_2 - b_1) - \alpha_{22}(kr\sigma_1^2 + 1)}{2kr\sigma_1^2 + 1}. \tag{41}$$

Substituting (41) into (39) and solving for α_{22} and α_{21}, we find (25) and (24). Substituting these expressions into (40), we obtain (23). The equilibrium effort $\mu = (\alpha_1 + \alpha_{21} + \alpha_{22})/k$ is then (16), so welfare $W^{TP} = (b_1 + b_2)\mu - k\mu^2/2 - r[(\alpha_1 + \alpha_{21})^2\sigma_1^2 + \alpha_{22}^2\sigma_2^2]/2$ is equal to (17).
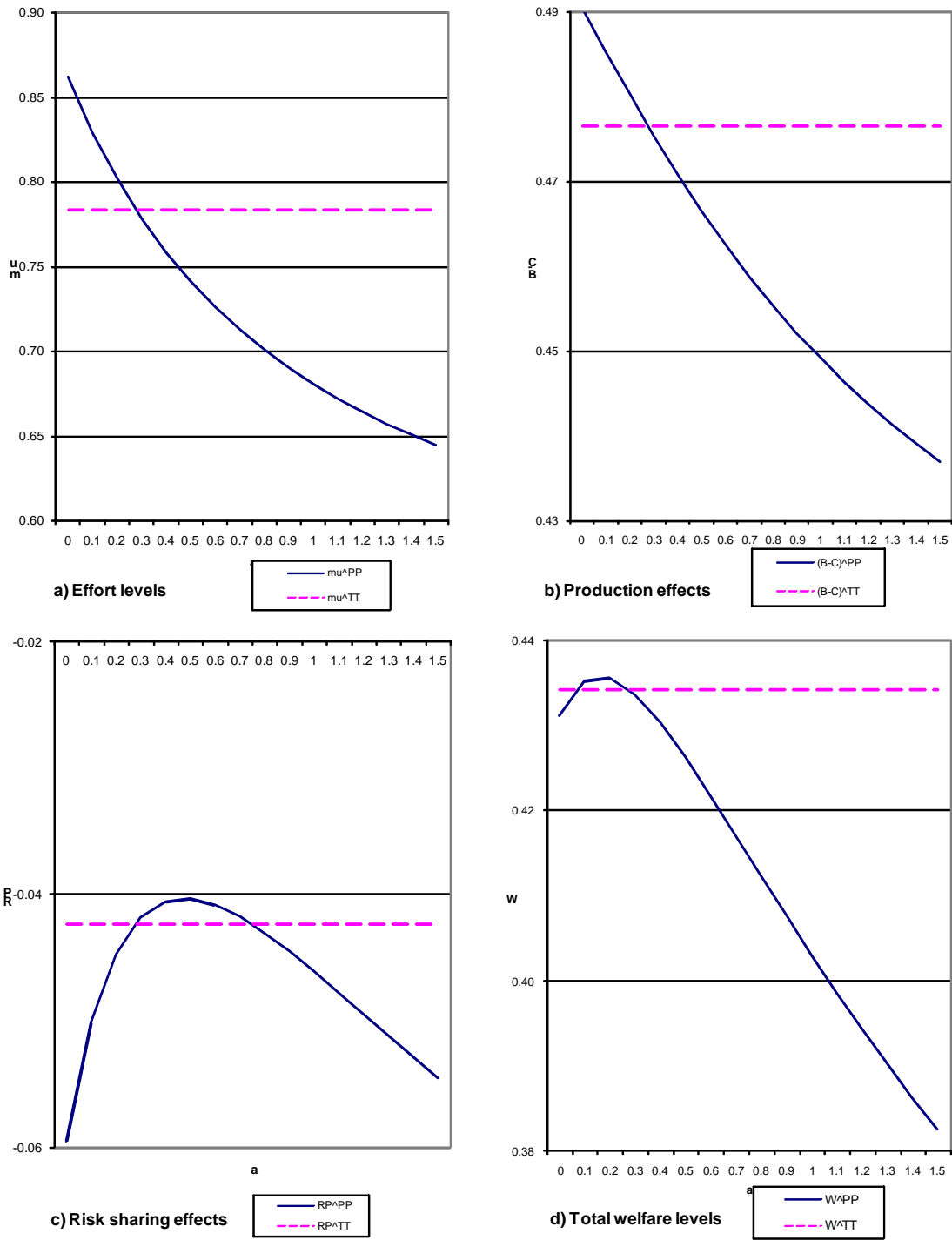
Figure 1. The four panels compare transparent (dashed line) and private (continuous line) contracting as a function of a, for an example with $\sigma_1 = 1$, $\sigma_2 = 0.4$, $b_1 + b_2 = 1$, and $r = k = 1$.