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Longevity gap, indexation and age-specific average pensions

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ABSTRACT
Studying the age-dimension of the probability distribution of pensions while assuming steadily rising real wages and time-invariant benefit-rules, two factors play important roles: (i) the weight of the wages in indexation of benefits in progress; (ii) the longevity gap. Factor (i) acts against relative depreciation of older benefits, while factor (ii) raises the share of higher benefits among older cohorts. Using an example and a model we show how the shape of the average benefit--age-curve depends on the relation between these two factors.

JEL codes: H55
Keywords: public pension system, longevity gap, indexation of pensions in progress, age-specific pensions

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Élettartamrés, indexálás és az életkortól függő átlagnyugdíj

SIMONOVITS ANDRÁS

ÖSSZEFOGLALÓ

A nyugdíjeloszlás életkortól függését vizsgáljuk, feltesszük, hogy a reálbérek tartósan emelkednek és a nyugdíjszabályok változatlanok. Két fontos tényezőre összepontosítunk: 1) a már megállapított nyugdíjak indexálásában a bérek súlya, 2) az élettartamrés. Az 1) tényező gátolja a korábban megállapított nyugdíjak relatív értékvesztését, a 2) tényező pedig a nagyobb nyugdíjak súlyát emeli a régebben megállapított nyugdíjakon belül. Egy példát és egy modellt használunk, hogy a két tényező együttesen hogyan hat az átlagnyugdíj és az életkor közti kapcsolatra.

JEL: H55

Kulcsszavak: tb-nyugdíjrendszer, élettartamrés, már megállapított nyugdíjak indexálása, életkortól függő nyugdíjak

Longevity gap, indexation and age-specific average pensions

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September 26, 2022

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Abstract

Studying the age-dimension of the probability distribution of pensions while assuming steadily rising real wages and time-invariant benefit-rules, two factors play important roles: (i) the weight of the wages in indexation of benefits in progress; (ii) the longevity gap. Factor (i) acts against relative depreciation of older benefits, while factor (ii) raises the share of higher benefits among older cohorts. Using an example and a model we show how the shape of the average benefit–age-curve depends on the relation between these two factors.

Keywords: public pension system, longevity gap, indexation of pensions in progress, age-specific pensions

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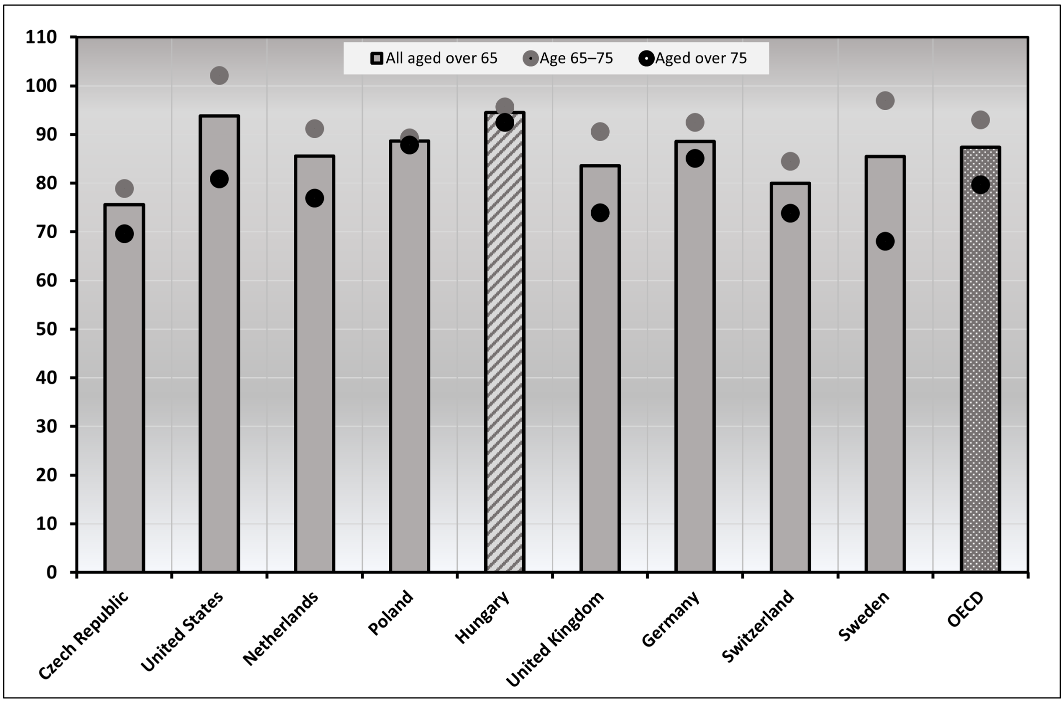
1 Introduction

Studying income inequalities among old-age citizens, in the bulk of developed nations, public pensions play a definite role. Pension inequality is a multi-causal process, including factors like the type of the benefit–contribution-link, the dynamics of generosity, etc. Confining our attention to the age-dimension of the inequality, we shall concentrate on the type of indexation (the weight of wages) and the size of the longevity gap (as a rule, poorer people die earlier). We shall create an example and a minimal model where the interaction between these two factors can quantitatively investigated.

A volume edited by Feldstein and Liebman (2002) already studied the distributional impact of Social Security (cf. also Brown (2001); Barr and Diamond, 2008, Chapter 7). The role of indexation of benefits in progress has received much less attention, though already Diamond (2004, p. 7. ftn. 24) had the following suggestion: while in the US the benefits in progress grow parallel with consumer prices, "... it would be better, on a revenue neutral basis, to have lower initial benefits that then grew faster (for example as a weighted average of prices and wages). This would help more the longer-lived than the shorter-lived but the effect on expected lifetime income distribution could be partially adjusted by changing the benefit formula" (see also Barr and Diamond (2008, Chapter 5).

Chetty et al. (2016) invigorated the interest in the longevity gap. OECD (2019, Chapter 7) contains a lot of new information on the issue in general, and the age-dimension in particular. Figure 1 displays the wildly diverging relative income positions of younger and older pensioners in various countries. For example, in Hungary and Poland there is hardly any difference between the two categories, while in the other countries, the older incomes, including pensions—more or less—are falling behind the younger ones.

Figure 1. The relative income position of younger and older pensioners in selected OECD countries



Source. OECD (2019) Table 7.1.

The early Hungarian studies on the redistribution of public pensions (Martos, 1994); Major and Martos, 2000) concentrated on the weak link between contributions and benefits, resulting in compressed distribution of benefits with respect to that of wages. Since then, the foregoing link has become much stronger and the phasing-out of wage-indexation of benefits-in-progress has given an outstanding role to the age dimension (Simonovits, 2018). D. Molnár and Hollós Marosi (2015) documented the positive correlation between pensions and life expectancy in Hungary. At the same time, Krémer (2015) observed a remarkable consequence: the pensions' distribution is much more symmetric than the wages' are, though the initial benefits are more or less proportional to wages.

Analyzing the impact of indexation on pensions, Simonovits (2020) already extended the analysis to the longevity gap. Continuing Krémer's observation, the present paper asks the following question: what is the shape of the conditional average benefit–age-curve? Using a simple, two-by-two class example, and a minimal cohort model, we determine the quantitative conditions making the average benefit–age-curve rising or declining. In contrast to Simonovits (2020), the paper does not draw any policy conclusion but one thing is clear: the foregoing curve may hide a complex two-dimensional probability distribution with respect to benefits and ages.

Among the rich literature, we only mention the closest sources. Sheshinski and Coliendo (2021) modelled the impact of the longevity gap on the income redistribution in the US Social Security presuming indexation to prices. Simonovits and Lackó (2021) analyzed the impact of the longevity gap on the redistribution within a mixed pension system, combining flat and proportional pillars in a stationary model without explicit indexation. Simonovits (2022) discussed the interaction of the longevity gap and the contribution cap.

The structure of the remaining part is as follows: Section 2 displays an example depicting the relation between younger/older and better-off/worse-off pensioners. Section 3 presents an annual cohort model, while Section 4 draws the conclusions. An Appendix A contains an empirical distribution of Hungarian male pensioners, Appendix B explains the background of the numeric calculations.

2 Example

This section presents the interaction of longevity gap and partial wage indexation in a very simple example. Pensioners live maximum two periods and there are only low and high benefits with index $i = L, H$. Their earnings in period $t = 0$ were $w_{\rm L} < 1 < w_{\rm H}$, their relative frequency are $f_{\rm L}, f_{\rm H} > 0$, $f_{\rm L} + f_{\rm H} = 1$. A proportional pension system is operated: $b = \beta w$, i.e., $b_{\rm L} < b_{\rm H}$, $\beta > 0$ being the accrual ratio. Due to our stationarity assumption, the survival probabilities of types L and H are denoted by $0 \le p_{\rm L} \le p_{\rm H} \le 1$. Calculating in real values, let G be the cumulated period growth coefficient. Benefit of type i remaining from period –1 was equal to $\beta w_i G^{-1} = b_i G^{-1}$, by now it grew by a factor G^ι. With notation $\theta = 1 - \iota$, their values are $b_i G^{-\theta}$, $i = L, H$. The structure of the retired population is given in Table 1.

Table 1. Two-dimensional distribution of pensions

Type	Younger (1)	Older (p_i)
Low earner ($f_{\rm L}$)	$b_{\rm L}$	$b_{\rm L}G^{-\theta}$
High earner($f_{\rm H}$)	$b_{\rm H}$	$b_{\rm H}G^{-\theta}$

Source: own calculation

The average of initial benefits and benefits in progress are respectively given by

$$\bar{b}_0 = f_\mathrm{L} b_\mathrm{L} + f_\mathrm{H} b_\mathrm{H} \qquad \text{and} \qquad \bar{b}_1 = \frac{f_\mathrm{L} p_\mathrm{L} b_\mathrm{L} + f_\mathrm{H} p_\mathrm{H} b_\mathrm{H}}{f_\mathrm{L} p_\mathrm{L} + f_\mathrm{H} p_\mathrm{H}} G^{-\theta}.$$

The former is less than the latter if and only if

$$(f_\mathrm{L} b_\mathrm{L} + f_\mathrm{H} b_\mathrm{H})(f_\mathrm{L} p_\mathrm{L} + f_\mathrm{H} p_\mathrm{H}) G^{\theta} < f_\mathrm{L} b_\mathrm{L} p_\mathrm{L} + f_\mathrm{H} b_\mathrm{H} p_\mathrm{H},$$

i.e., introducing the L-to-H benefit ratio $\alpha = b_\mathrm{L}/b_\mathrm{H}$ and the L-to-H ratio of survival probabilities $\pi = p_\mathrm{L}/p_\mathrm{H}$, the inequality simplifies to

$$G^{\theta} < \frac{f_\mathrm{L}\alpha\pi + f_\mathrm{H}}{(f_\mathrm{L}\alpha + f_\mathrm{H})(f_\mathrm{L}\pi + f_\mathrm{H})} = \lambda.$$

By the sum inequality of Chebyshev, $\lambda > 1$ (Simonovits, 1995). With $G > 1$ and $\pi > 1$, for $\iota = 1$ (wage indexation), the inequality strictly holds. As the wage indexation weight decreases, the left hand side increases. There are two cases: (a) the inequality holds even for $\iota = 0$: $G < \lambda$, then the average benefits of the older is always greater than that of the younger. (b) $G \geq \lambda$, then there exists a wage indexation weight $\iota_G \in [0, 1)$ such that when the two averages are equal, for lower/higher wage indexation weight, the average benefits of the older is greater/less than that of the younger: the impact of the gap or the less than full wage indexation dominates, respectively.

Table 2 illustrates the impact of the gap and wage indexation weight on the old and the young benefits. Assume two retirement periods of lengths 15–15 years, choose H's survival probability $p_\mathrm{H} = 1/2$, and relative frequencies $f_\mathrm{L} = 2/3$ and $f_\mathrm{H} = 1/3$, new benefits $b_\mathrm{L} = 1/4$ and $b_\mathrm{H} = 1$. Two parameter values change, the gap: $\Gamma = 15(p_\mathrm{H} - p_\mathrm{L})$ and the wage indexation weight ι, while the annual growth rate $G^{1/15} - 1 = 0.02$ is fixed. For the lack of gap and full wage indexation in the NW corner of Table 2, the two averages are the same. As we depart from the benchmark, the ratio of the two averages varies: the maximum is achieved at the maximal gap and wage indexation (SW corner): 1.333, the minimum is achieved at the no gap and price indexation (NE corner): 0.743. As a summary, a rise in the longevity gap as well as in the wage indexation weight raise the old-to new benefit ratio.

Table 2. Longevity gap, wage indexation weight and the ratio of the two benefits

Longevity gap (year) $\Gamma = 15(p_\mathrm{H} - p_\mathrm{L})$	Wage indexation $\iota = 1$	Mixed indexation $\iota = 0.5$	Price indexation $\iota = 0$
0.00	1.000	0.862	0.743
2.25	1.125	0.970	0.836
4.50	1.333	1.149	0.991

Source: own calculation

Finally, Table 3 displays the L-to-H ratio of standard deviations for the foregoing 3×3 cases. For the zero gap case, the standard deviation ratios are equal to the benefit ratios, but by increasing the gap and decreasing the wage indexation weight, the standard deviation ratio decreases.

Table 3. Longevity gap, wage indexation weight and the ratio of L-to-H standard deviations

Longevity gap (year) $15(p_\mathrm{H} - p_\mathrm{L})$	Wage indexation $\iota = 1$	Mixed indexation $\iota = 0.5$	Price indexation $\iota = 0$
0.00	1.000	0.862	0.743
2.25	0.972	0.838	0.722
4.50	0.833	0.718	0.619

Source: own calculation

3 Cohort model

In this Section, first we enumerate the assumptions of the cohort model, then derive the formulas, and finally using stylized data, we fill them with numbers.

3.1 Assumptions

First of all, we present the basic assumptions of the model. Obviously, in every dimension the reality differs from the model, but with the help of the model, we are able to prove meaningful results.

A1. Unisex population: no differentiation between males and females.

A2. Stationary population: every year the number of persons born is the same, and nobody dies before reaching retirement age.

A3. The post-retirement age-specific mortality is a decreasing function of life earnings, but is independent of pensions.

A4. Everybody retires at the same age, regardless of wage and calendar year.

A5. The growth rate of the average real wages is time-invariant.

A6. The distribution of earnings is time-invariant.

A7. The initial benefits are proportional to the lifetime earnings.

A8. Every year, the real value of benefits in progress increases by a given share of the growth rate of the average earnings.

A9. The elimination of the inflation does not influence the real value of the benefits.

3.2 Formulas

There is no inflation, we calculate at constant prices. Calendar years are indexed by variables $t = \ldots, -1, 0, 1, \ldots$. We study the situation in year 0, but assume that the final wage distribution $E(w)$ with a density function $e(w)$ applies before and after 0, only each element is multiplied by $g \geq 1$ in every year. The pensioners of year 0 are characterized by two numbers: the number of years elapsed after retirement: $a = 0, 1, \ldots, A-1$ with maximum A; and wage w if her final earning was equal to wg^{-a}. Then the person's initial benefit a years earlier was $b_{0,-a}(w) = \beta w g^{-a}$, annually multiplied by g^{ι}, where $\iota \in [0, 1]$ is the wage indexation weight, therefore, the corresponding benefit in year 0 is equal to

$$b_a(w) = \beta w g^{-a} g^{\iota a} = \beta g^{-\theta a} w, \qquad \theta = 1 - \iota \geq 0.$$

Assuming stationary population, where each cohort is represented by a large number of persons, the survival probability after year a is denoted by $p_a(w)$, being a decreasing function of a, and an increasing function of w.

In year 0, let us denote $f_a(b)$ the density function of pensions of those who retired a years ago, where $b_{\mathrm{m}} > 0$ stands for the minimum benefit. First we replace the exit–age curve by a step function: no risk. To apply the well-known theorem, we need the inverse function of wage–benefit. Introducing notation $\kappa_a = \beta^{-1}g^{\theta a}$, the corresponding wage is given by $w = \kappa_a b_a$ and assuming that the benefit–wage function has a derivative $\psi'(w) = 1/\kappa_a$ – which is constant. Then the riskless benefit density function is equal to

$$\tilde{f}_a(b_a) = e(\kappa_a b_a)\kappa_a.$$

Second, we get rid of the assumption of no risk: during a years a worker of index w survives the exits with probability $p_a(w/\kappa_a)$ and the actual density function is the product of the survival probability and the riskless density function in the appropriate interval:

$$f_a(b_a) = p_a(\kappa_a b_a)\tilde{f}_a(b_a) = \kappa_a p_a(\kappa_a b_a)e(\kappa_a b_a).$$

We shall characterise the age-dimension by the age-dependent average benefit of pensioners in year 0, who retired a years ago:

$$\bar{b}_a = \int_{b_{\mathrm{m}}}^{\infty} b_a f_a(b_a)\, db_a.$$

Substituting the density function into the expected value formula yields the age-dependent average benefit:

$$\bar{b}_a = \kappa_a \int_{b_{\mathrm{m}}}^{\infty} b_a p_a(\kappa_a b_a)e(\kappa_a b_a)\, db_a.$$

3.3 Parameterization

Having outlined the model, now we are trying to obtain sensible numerical results on the conditional benefit–age-function. Since we have made a large number of simplifying assumptions, there is not much use to consider real-life data. At the start we give the initial wage distribution by a Pareto distribution of power σ:

$$E(w) = 1 - w^{\sigma}/w_{\mathrm{m}}^{\sigma}, \qquad \text{where} \qquad \sigma > 1.$$

The corresponding density function is

$$e(w) = \sigma w^{\sigma-1}/w_{\mathrm{m}}^{\sigma}.$$

To normalize wages, we assume that the expected wage is equal to 1:

$$\mathbf{E}w = \int_{w_{\mathrm{m}}}^{\infty} we(w)\, dw = 1.$$

Then $w_{\mathrm{m}} = (\sigma - 1)/\sigma$.

Second, the survival probability function of age and of wage is approximated by a power function used by Sheshinski and Coliendo (2021):

$$p_a(w) = 1 - (a/A)^{\gamma+\psi w}, \qquad \gamma, \psi > 0.$$

We shall choose the values of (γ, ψ) so that the difference between the life expectancies of the highest and the lowest deciles be realistic: $\gamma = 1.1$ és $\psi = 0.3$. Figure 2 shows the survival functions of the highest and the lowest deciles. We mention that the numbers of years spent in retirement by the lowest and the highest deciles are equal to $L_1 = 17.2$ and $L_{10} = 20.6$ years, respectively.

Figure 2. The survival functions of the highest and the lowest deciles

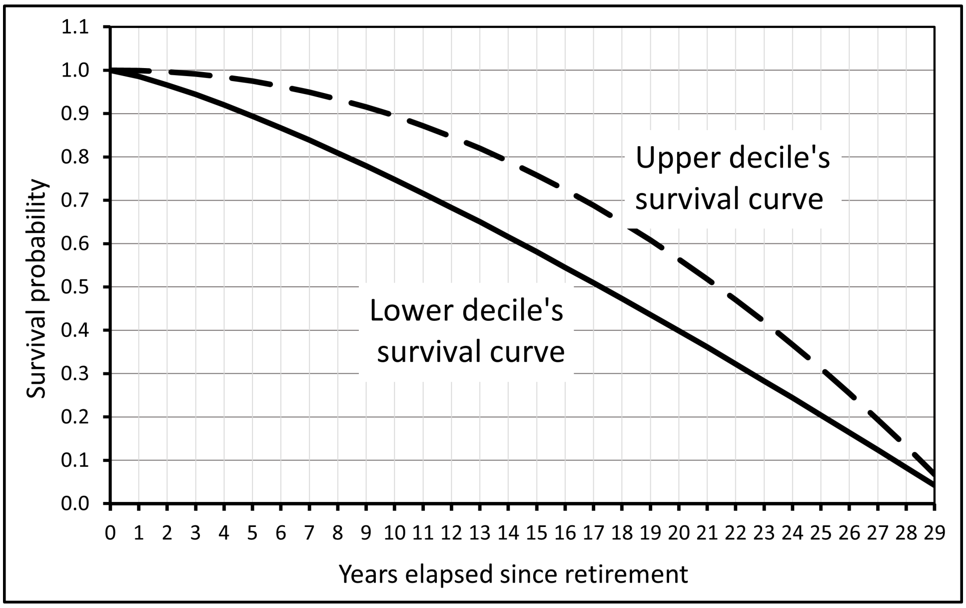
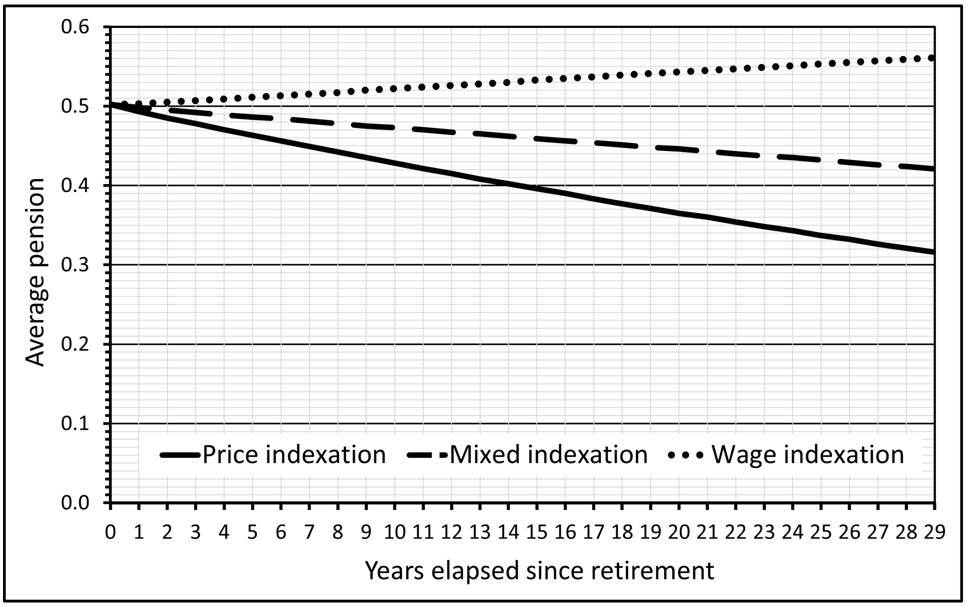


Figure 3 displays three average benefit–age-curves in year 0 for three wage indexation weights. We present the average benefits in terms of the gross average wages. For price indexation: 0.429; for mixed indexation: 0.472 and for wage indexation: 0.522. The first and the second curves are declining, the third is increasing.

Figure 3. Average benefit–age-curves



4 Conclusions

With the help of a simple example and a minimal cohort model we have obtained numerical estimates on the impact of longevity gap and of the wage indexation weight on the age-dependent pension inequality. If there is no gap and the wage indexation is full, then the age-dependent average benefit is constant. In general, however, two tendencies counteract each other: the greater the gap, the more steeply rises the indicator; the lower the wage indexation weight, the more steeply declines the indicator. From the point of view of welfare maximization, neither tendency is socially optimal. A fair solution (Diamond, 2004 cited above) is to add a flat benefit to the proportional one, analyzed by Simonovits and Lackó (2022).

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Appendix A. Empirical data

Table A.1 is the only empirical table in the paper, showing the joint distribution of Hungarian male pensioners with respect to benefits and age, with due normalization. (If I had access to detailed data, I could have constructed a more convincing contingency table.) Apart from the extreme age-groups (males below 64 and males above 95), the other age-groups span five years. We do not know the average ages of the youngest and of the oldest age-groups, we arbitrarily choose them as 62 and 97 years. The common width of benefit classes is equal to 15.4% of the average benefits, and we represent them by their middle points. We do not know the minimal and the maximal averages, either, we arbitrarily choose them as 15.3 and 230.8% of the average benefits, respectively.

Table A.1. The joint distribution of retired Hungarian males in 2019 by benefits and ages

Relative w.r.t. average benefit %	–64	65–69	70–74	75–79	80–84	85–89	90–94	95+	Average age, year	Relative frequency %
	Age groups									
7.4	4.0	0.9	1.0	1.2	1.4	1.2	0.9	0.9	68.0	1.7
23.0	25.6	1.8	1.2	0.9	0.6	0.5	0.3	0.1	63.3	6.6
38.3	17.0	3.7	2.5	1.8	0.9	0.2	0.4	0.4	64.9	5.7
53.6	14.1	9.9	7.7	6.1	3.8	2.3	2.9	3.7	68.0	8.9
68.9	9.6	15.4	15.4	15.8	13.9	11.2	8.8	7.3	71.3	13.8
84.2	5.7	14.4	14.3	16.4	19.4	18.1	15.6	12.2	72.9	13.3
99.5	3.6	12.3	12.2	13.5	15.8	16.7	17.0	16.2	73.3	11.1
114.9	7.1	10.0	10.1	10.9	12.8	13.6	13.9	16.5	72.3	10.0
130.2	3.4	7.6	8.2	9.2	10.1	10.3	11.4	12.1	73.0	7.4
145.5	2.4	5.9	6.7	7.9	8.4	7.7	9.0	9.5	73.3	5.9
160.8	1.8	4.5	5.2	6.3	5.6	6.5	7.2	7.8	73.3	4.5
176.1	1.4	3.2	3.9	4.1	3.1	4.5	4.8	5.6	72.8	3.2
191.4	1.2	2.5	3.1	2.4	1.9	3.0	3.2	3.3	72.1	2.3
206.7	1.0	1.9	2.5	1.4	1.1	1.8	1.9	1.7	71.3	1.7
222.1	0.7	1.4	2.0	0.8	0.5	1.1	1.2	1.2	70.8	1.2
237.0	1.5	4.6	3.9	1.4	0.7	1.3	1.4	1.5	69.5	2.8
Population weight %	22.2	28.2	20.4	14.8	8.2	4.4	1.5	0.3	–	100
Average benefit	*66.2*	*107.9*	*112.4*	*108.1*	*107.5*	*114.8*	*117.9*	*120.6*	*100.0*	–
Relative standard deviation, %	0.775	0.484	0.461	0.410	0.369	0.368	0.357	0.346	–	–

Source: Constructed on the basis of Table 11.6, CSO (2020).

The age-specific distributions are given by the columns $-64, \ldots, , 95+$, the sum of the entries of each column is approximately equal to 100%. The last but one column contains the average age of the benefit classes, the last column presents the corresponding shares. For example, in the row 99.5, the average age is equal to 73.3 years, the corresponding share is equal to 11.1%. Within a given row, the other numbers represent the corresponding weight of the given year-group. For example, the share of the group 80–84 is equal to 15.8% in that row.

From our point of view, the most important observations are as follows: (a) the relative average benefits do not decrease with rising age (italicized row): for age-group 65–69, it is equal to 107.9% of the average benefits, then fluctuates it rises from 114.8 (age-group

85–89) to 120.6% (age-group 95+); (b) the relative standard deviation decreases with age and (c) the benefit-specific average age increases until benefit class 160.8% and only then decreases.

Appendix B. The backgound of the numerical calculations

Appendix B presents the discrete approximation of a Pareto distribution, needed at numerical integration. The i-th upper endpoint is denoted by W_i, defined by

$$E(W_i) = 1 - W_0^\sigma / W_i^\sigma, \qquad i = 1, \dots, I, \qquad \text{where} \qquad \sigma \geq 2, \tag{B.1}$$

and W_0 stands for the minimum wage. Dividing the distribution $E(\cdot)$ into equal parts:

$$E(W_i) = \frac{i}{I} = \delta i. \tag{B.2}$$

Substituting the power function (B.1) into (B.2) yields

$$1 - W_0^{-\sigma} W_i^{-\sigma} = \delta i.$$

Solving the implicit equation for the upper endpoint:

$$W_i^{-\sigma} = W_0^{-\sigma}(1 - i\delta), \qquad \text{i.e.,} \qquad W_i = \frac{W_0}{(1 - i\delta)^{1/\sigma}}. \tag{B.3}$$

It would be useful to approximate the middle point of the i-th interval by the geometric mean of the two subsequent endpoints in (B.3):

$$w_i = \sqrt{W_{i-1} W_i} \qquad i = 1, \dots, I - 1 \qquad \text{and} \qquad w_I = \frac{\sigma}{\sigma - 1} W_{I-1}. \tag{B.4}$$

Substituting the endpoints into the middle point formula (B.4) yields a new closed formula:

$$w_i = W_0[(1 - (i - 1)\delta)(1 - i\delta))^{-1/(2\sigma)}. \tag{B.5}$$

It is worth defining W_0 the average wage be equal to 1:

$$\sum_{i=1}^{I} f_i w_i = 1.$$

It is easy to see that $W_0 = (\sigma - 1)/\sigma$.

At the numerical calculations we shall work with deciles: $I = 10$ and power-index $\sigma = 2$. Table B.1 displays the limits and the middles:

Table B.1. End- and middle points

Decile index i	Upper end-points W_i	Middle point w_i
1	0.527	0.513
2	0.559	0.543
3	0.598	0.578
4	0.645	0.621
5	0.707	0.676
6	0.791	0.748
7	0.913	0.850
8	1.118	1.010
9	1.581	1.330
10	–	3.162

Source: own calculation